MELTING POT OR SALAD BOWL: THE FORMATION OF HETEROGENEOUS COMMUNITIES

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Abstract

Relatively little is known about what determines whether a heterogenous population ends up in a cooperative or divisive situation. This paper proposes a theoretical model to understand what social structures arise in heterogeneous populations. Individuals face a trade-off between cultural and economic incentives: an individual prefers to maintain his cultural practices, but doing so can inhibit interaction and economic exchange with those who adopt different practices. We find that a small minority group will adopt majority cultural practices and integrate. In contrast, minority groups above a certain critical mass, may retain diverse practices and may also segregate from the majority. The size of this critical mass depends on the cultural distance between groups, the importance of culture in day to day life, and the costs of forming a social tie. We test these predictions using data on migrants to the United States in the era of mass migration, and find support for the existence of a critical mass of migrants above which social structure in heterogeneous populations changes discretely towards cultural distinction and segregation.

JEL codes: D85, J15, Z12.

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1 Introduction

Ethnic, linguistic, and religious heterogeneity are associated with a variety of politico-economic problems, including low growth, low provision of public goods, and conflict. Yet the effects of diversity are not uniform: some heterogenous populations manage to avoid societal conflict and are economically successful. In some cases, diversity can even have positive effects on growth, productivity, and innovation.[1] Seeking to explain this puzzle, recent work finds that social cohesion plays an important role in how well a population 'deals' with its heterogeneity. For example, more segregated populations are found to have lower quality of governance, lower trust between citizens, and worse education and employment outcomes, compared to similarly heterogenous populations that are better integrated.[2]

Despite its clear importance, relatively little is known about what determines whether a heterogenous population ends up in a cooperative or divisive situation. In this paper we propose a model that allows us to study this question. We ask, first, what social structures can arise when society is composed of heterogeneous groups, and second, what features influence which social structure arises in a particular environment. Understanding this is crucial in an increasingly mobile world in which immigration and social cohesion are frequently at the forefront of political agendas.[3]

In our model, individuals from two distinct groups – natives and immigrants – living side-by-side, make choices over activities to engage in, and who in the community to interact with. Some activities, such as language choice, are 'cultural actions', in the sense that each group will start with an *ex ante* preferred language, and there is a cost to switching to any other choice. Other activities are 'non-cultural': there is no group-specific reason why one activity is more costly to engage in than another.[4] For example, there may be no group-specific reason why one sport should be more costly to engage in than another.[5] Interaction provides opportunities for

[1]Easterly and Levine (1997), Alesina et al. (1999), Montalvo and Reynal-Querol (2005), Ashraf and Galor (2013), and Rasul and Rogger (2015), amongst others.

[2]See for example Alesina and Zhuravskaya (2011) and Cutler and Glaeser (1997). Firm and team studies suggest that environments that bring together heterogeneous skills and ideas, at the same time as fostering cooperation, are associated with positive effects of diversity on innovation and productivity. At the national level Ashraf and Galor (2013) show a hump-shaped relationship between genetic diversity and growth, consistent with their hypothesis that diversity is beneficial for innovation and production but can also be costly when it reduces cooperation and increases disarray within a society. See Alesina and La Ferrara (2005) and Alesina et al. (2013) for further discussion and references.

[3]Europe provides an example of a variety of policies designed to integrate immigrants and perhaps also influence voters. For example, in France, in 2010, the senate voted $246 - 1$ to ban the full Islamic veil, with the French President arguing it was not consistent with French identity. In the UK, in 2013, the government announced plans to increase English language requirements to ensure migrants were 'able to integrate into British society'.

[4]In the model presented in the paper an individual chooses only two actions, a cultural action and a non-cultural action. In Online Appendix B we allow for multiple cultural and non-cultural actions.

[5]Of course some groups have ties to particular sports and so which activities are non-cultural and cultural will depend on the groups. A word of caution: a result of this paper is that non-cultural activities can in equilibrium become associated with a

economic exchange, and so individuals can benefit from forming social ties with others in the population. Since interaction requires some degree of commonality of actions, we assume that the benefit of a link is increasing in the number of shared activities, with a fixed (per-link) cost of formation. The fundamental trade-off is that an individual prefers to maintain his cultural practices but doing so can hinder opportunities for interaction and exchange with those who adopt different practices.[6]

We find that only three classes of social structure are possible in (Nash) equilibrium: assimilation, segregation, and multiculturalism. In assimilation equilibria, all individuals engage in the same activities (cultural and non-cultural), and interact with everyone. In segregation equilibria, all individuals play their type-specific cultural activities, and form social ties only with those of the same type. In multicultural equilibria, individuals play their type-specific cultural activities, but all coordinate on common non-cultural activities, and this allows individuals to form ties with all others, regardless of type. The existence of shared non-cultural practices allows interaction between disparate groups who maintain distinct cultures.

What kind of environments sustain these three outcomes? The key pattern we identify is that social structure in heterogeneous populations changes discretely in the share of the minority. When the minority (immigrant) group is small relative to the majority, assimilation occurs. Intuitively, restricting social interaction within such a small group is not desirable and an immigrant does better by paying the cost of switching culture and being absorbed into a much larger group. Multiculturalism and segregation are equilibria only once the share of the minority group in the population exceeds particular (different) thresholds. Above a certain threshold, there is a large enough 'critical mass' of immigrants that if the group maintains its distinct culture then, for any immigrant, the cost of switching culture outweighs the benefits of increased interaction. This threshold result is due to complementarities: maintaining a distinct minority culture is only 'worth it' if others do so, therefore either no one maintains the immigrant culture or a large group of immigrants does.

The location of these thresholds, above which segregation and multiculturalism are equilibria, depends on three parameters: (*i*) cultural distance; (*ii*) the importance of culture in

particular group. Thus many of the examples that spring to mind that associate sport or other activities with a particular group may in fact be an equilibrium outcome and not a result of some *ex ante* preferences.

[6] In an example of this, Algan et al. (2013) find an economically significant trade-off faced by Arabic parents in France between attachment to their own culture (in their study, the desire to pass on an Arabic name) with the future economic performance of their children in the form of work-related penalties to having an Arabic name. This is an important trade-off and is present in varying forms in Kuran and Sandholm (2008), Lazear (1999), Bisin et al. (2011), Carvalho (2013b), and Carvalho (2013a), amongst others.

everyday life; and (*iii*) the cost of forming a social tie. When the cultural distance between immigrants and natives is larger, the cost of switching cultural actions is relatively higher. This reduces the threshold immigrant share at which multiculturalism and segregation – equilibria where immigrants retain their own culture – become possible. In contrast, a higher importance of culture in daily life makes having a common culture more important for interaction, and so it is harder to sustain multiculturalism (the threshold on multiculturalism rises). To illustrate, if social interaction involving alcohol is ubiquitous, and an immigrant's culture prohibits alcohol, then this makes it difficult for the immigrant to both maintain his cultural practices and integrate with natives, and thus he is forced to choose. Finally, higher costs of forming a link lower the threshold on segregation. Intuitively, since the costs of assimilation come from switching culture, while the benefits come from improved interaction with natives, a higher cost of link formation lowers the relative benefits of assimilation and makes segregation easier to sustain.

We extend our model to account for multiple generations, where we consider cultural transmission from parents to children.[7] Considering transmission allows us to see which equilibria are most likely in the long run and how robust our static results are. We find the same general pattern as the static model: the three classes of social structure can exist in the long-run; there is a distinct threshold for each structure, based on the share of immigrants in the population; and these thresholds depend on cultural distance, the importance of culture in everyday life, and the cost of forming a link.

We test these predictions using census data on immigrant populations in the United States at the beginning of the 20th Century. During this period large numbers of immigrants arrived in America, radically altering the national make-up of the country. This is exactly the kind of situation our framework seeks to understand. Indeed, the three distinct forms of social structure we find are evident in scholarly discussion of communities in the United States following mass migration (Gordon, 1964). Early scholars argued that a single culture would prevail: the so-called 'melting pot'. As it became clear that not all communities assimilated, but some instead 'retained distinctive economic, polical and cultural patterns long after arriving in the United States', segregation became a major concern (Bisin and Verdier, 2000). However, there was also discussion of a 'third way' – 'the salad bowl' – where immigrants could become 'American' and integrate whilst maintaining some cultural distinction. This period in history has the added

[7] Formally we use stochastic stability techniques, see Young (1993) and Kandori et al. (1993). Children inherit their parent's culture, and choose how to best respond to the existing actions of the population, but occasionally 'experiment' by playing suboptimal actions.

benefit of furnishing us with data on a large number of heterogeneous communities with varying immigrant group sizes, which is vital to any chance of observing thresholds.

If our framework captures the key forces driving community formation, then we should expect behavior in heterogeneous populations to exhibit the threshold patterns predicted by (and central to) the model. We test two key predictions of the model: that a threshold should exist in community behavior, and that the location of this threshold should depend on cultural distance from the existing population of the US. These predictions also allow us to separate our model from explanations based on selection, which do not suggest the discontinuities between group size and community outcomes that are critical to our model. We test our predictions on the decision of immigrants to acquire the English language or not (a cultural action), and on the decision to in-marry (a partial measure of interaction). We find sharp and significant thresholds in behavior when an immigrant group forms around one-third of the community: above this threshold English acquisition in the immigrant group falls by almost a half, from 90% to 50%, and in-marriage rises by a third from 55% to 75%. Using data on linguistic distance between the language of the immigrant group and English, we additionally show that, as predicted, when cultural (linguistic) distance is higher, the estimated threshold for segregation and multicultural outcomes is lower.

The literature examining choice of culture highlights the variety of outcomes that can arise in heterogenous populations. In seminal work in economics on cultural transmission, Bisin and Verdier (2000) examine the persistence of different traits in a mixed community even in the long run.[8] Iannaccone (1992), Berman (2000), and Carvalho (2013b) study the stability of costly and restrictive cultural practices within religious groups in heterogenous societies.[9] Kuran and Sandholm (2008) examine the convergence of cultural practices when diverse groups interact. The question we ask in this paper is different. We want to understand *selection* between various outcomes.[10] That is, when does one of these social structures emerge rather than another?

It is important to highlight the two features of the theoretical framework that make it rich

[8]See also Bisin et al. (2004) for empirical work on this topic. A key finding in Bisin and Verdier (2000), that smaller groups may exert more effort in passing on their culture to their children, sounds at odds with our finding that smaller groups are more likely to assimilate. This is not the case. When groups continue to maintain different practices in equilibrium we find that smaller groups must put in more 'effort' in the form of maintaining higher diversity of practices in order to sustain segregation. See the model of action choice along more than two dimensions in Online Appendix B. Bisin and Verdier (2000) also point out that effort does not necessarily relate monotonically to outcomes: a small group could put in lots of effort to pass on their culture but it may still die out, depending on the cost function.

[9]Berman (2000) and Iannaccone (1992) model religion as a club good with 'extreme' cultural practices as a means of taxing other goods to increase contribution to the religious good. This is slightly different to the framework presented in our paper and the other papers cited, which draw on the Akerlof and Kranton (2000) model of identity.

[10]We are reassured that our framework permits, as equilibria, outcomes consistent with those that are studied in detail in these papers.

enough to produce the different social structures. The first is the interaction between choice of behavior and choice of interaction, discussed further below. The second is the novel introduction of non-cultural actions, which play an important role in uniting or dividing communities. Adoption of a common non-cultural action is necessary for the existence of multicultural equilibria. At the other end of the spectrum, rather than bridging the gap between groups, non-cultural actions can also be used to divide them. We find a subclass of segregation equilibria in which immigrants not only retain distinct cultural practices but also create 'new diversity' by adopting deliberately different non-cultural activities from natives. Such polarization of non-cultural practices occurs in order to maintain segregation by raising the cost of interacting with the other group. This extreme form of segregation occurs when culture is relatively unimportant in everyday life.

Close to our question, in pioneering work, Lazear (1999) studies the choice of whether or not to adopt the same language or culture.[11] In Lazear (1999) and Carvalho (2013a), agents choose a cultural practice, but take interaction as given. A second literature models link formation in order to study the important concept of homophily - the tendency for similar individuals to be linked. In this case individuals choose interaction but take behavior as given (Currarini et al., 2009; Bramoullé et al., 2012; Currarini and Vega-Redondo, 2011). We take a novel approach which, importantly, addresses both these choices (choice of practices and choice of interaction) within a single tractable framework.[12] Our alternative approach allows payoffs from links formed to depend not only on *ex ante* heterogeneity but also on the action choices of both partners.[13] This results in a broader range of social structures and comparative statics.

Our theoretical approach contributes to a literature on network formation where individuals play coordination games across links, in particular Jackson and Watts (2002) and Goyal and Vega-Redondo (2005). They show that in homogenous populations, under equilibrium refinement, total integration and social conformism always prevail. We show that this is not true for heterogenous populations in which different individuals have preferences for coordinating on different activities. Our analysis of network formation in heterogeneous populations enables us to study the important question of why different social structures prevail and when.

[11] See also Eguia (2013) who examines this from the perspective of discrimination.

[12] In a different framework, Bisin and Verdier (2000) highlight the importance of choice of social interaction and make segregation effort a choice. However, they do not analyse resulting levels of segregation since their focus is whether diverse cultural traits persist. Also Carvalho (2013b) examines a choice of segregation in the decision to veil or not.

[13] Or, to put it the other way around, payoffs from actions taken to depend not only on *ex ante* heterogeneity but also on who one linked with and their choice of action.

Card et al. (2008) and Chay and Munshi (2013), like us, test empirically for the presence of a threshold in community behavior that depends on the size of the minority population, where the location of this threshold is *a priori* unknown. Card et al. (2008) look at local migration and Chay and Munshi (2013) at local migration and voting behavior. In contrast, our interest is in social interaction and convergence (or not) of behaviors within the community. Together with Card et al. (2008) and Chay and Munshi (2013), our findings highlight the importance of looking for these kind of threshold patterns when considering community-related behavior. Our findings are also in line with recent work on the US by Abramitzky et al. (2014), Fouka (2014), and Fulford et al. (2015), which suggests that environmental features were significant in determining how culture evolved in 19th and early 20th century United States.

In Section 2 we present the basic model. Section 3 characterizes the Nash equilibria for a single generation and provides comparative statics. Section 4 develops the multiple generation framework. Section 5 provides empirical evidence from communities in the United States in the age of mass migration supporting the predictions of the model. The final section discusses implications for government policy and welfare and concludes.

2 The Single Generation Model

First we present the framework for choice of action, then introduce social interaction, and finally we summarize the payoffs.

2.1 Culture

A population (or community) consists of a set of individuals $i \in \{1, 2, \ldots, n\}$. Each individual i is endowed with a type, $k \in \{M, m\}$, which is common knowledge. We refer to the majority M types as 'natives', and the minority m as 'immigrants' (although many other interpretations are clearly possible). There are $n_M \in \mathbb{N}$ individuals of type M and $n_m \in \mathbb{N}$ individuals of type m, where $n_M, n_m \geq 2$, $n = n_M + n_m$, and we assume $n_M \geq n_m$.[14]

To illustrate the model with an example, consider the population to be a neighborhood. An immigrant group has moved into the neighborhood from a different country and has come with different cultural practices to the native group. We denote the cultural practices associated with

[14]We use the terms 'native group' and 'immigrant group' as an illustration. Of course we need not always consider the native group as the majority group, for example Aboriginal populations of Australia and Native American populations of the United States.

the native group by the action x^M and the cultural practices associated with immigrant group by the action x^m.[15] Cultural practices could be activities, such as type of food eaten or language spoken, or behaviors, for example concerning education, gender, or marriage. Since cultural practices are, in themselves, simply activities and behaviors, this leads to an observation: when immigrants move to a new country they can, if they wish, adopt the cultural practices of the native group (or vice versa). We refer to this as switching culture. While it is possible to switch culture, there is a cost c from doing so. This is how we define culture: an individual chooses an action x_i from the set $\{x^M, x^m\}$, where it is possible to adopt the action associated with the other group, but at a cost.[16]

Berry (1997) describes cultural changes that occur when groups with different practices share the same environment as ranging 'from relatively superficial changes in what is eaten and worn, to deeper ones involving language shifts, religious conversions, and fundamental alterations to value systems'. Regarding religion, Iannaccone (1992) highlights that 'people can and often do change religions or levels of participation over time'. The cost to switching culture can arise for a variety of reasons. There may be fixed costs involved, such as learning a new language, or participating in unfamiliar activities. Alternatively cultural practices may be considered valuable in their own right.[17] Culture can be so deeply entrenched that individuals find it psychologically costly to adhere to behaviors or attitudes that differ from the culture one has grown up with.[18] We reduce these different possibilities to a single cost, c, in line with previous work on culture choice (Akerlof and Kranton, 2000; Bisin and Verdier, 2000). The magnitude of c is interpreted as a measure of cultural distance.

It is rather extreme to assume that all activities going on in this neighborhood are necessarily associated with type or culture. For example, while religion often prescribes or prohibits certain activities, religions rarely impact all aspects of everyday life. Religions rarely proscribe what sports should be played. In a neighborhood with different religious groups it may be no more

[15]We model the practices that define a group's culture by a single action. We enrich this model of culture by allowing for multiple dimensions of culture in Online Appendix B. The main results remain.

[16]The modeling of culture and a cultural group presented here is consistent with the introduction of identity into economic modeling by Akerlof and Kranton (2000). Note that in our framework an individual chooses one action or the other. For some practices such as language, however, an individual might continue to speak his group's language, but also learn the language of the other group. The results will hold provided that the relative benefit of learning the native language increases the more members of the immigrant group there are that learn it. This would be the case, for example, if interaction and exchange among individuals and groups occurs more and more often in the native language as more immigrant group members learn it.

[17]Algan et al. (2013) estimate that the utility an Arabic parent in France gets from passing an Arabic name to their child is equivalent to a 3% rise in lifetime income of the child. See also Bisin and Verdier (2000) and Kuran and Sandholm (2008) for examples.

[18]See Berry (1997) for a review and further references on psychological and sociocultural costs. This framework does not incorporate group penalties, but such an assumption could be incorporated.

costly (*ex ante*) for a member of either religious group to pick one sport as compared to another. Which actions are specific to culture and which are not may vary by each particular situation. Non-cultural actions are modeled by an individual i's choice of action $y_i \in \{y^A, y^B\}$ for which there is no associated type-specific cost.[19] A word of caution: a result that we will highlight later in the paper is that activities and practices that *ex ante* have nothing to do with type and culture (for example, sports), can in equilibrium become associated with a particular group. Thus many of the examples that spring to mind that associate sport or other activities with a particular group may in fact be an equilibrium outcome and not a result of some *ex ante* preferences.

To summarize the modeling of culture, each individual has a given type, M or m. Each individual i of type $k \in \{M, m\}$ chooses an action set (x_i, y_i), where $x_i \in \{x^M, x^m\}$, $y_i \in \{y^A, y^B\}$. There is no type-specific cost associated with the non-cultural action y_i whilst the cost of cultural action x_i is

$$c_k(x_i) = \begin{cases} 0 & \text{if } x_i = x^k \\ c & \text{if } x_i \neq x^k \end{cases}$$

2.2 Social Interaction

Were this the end of the model, individuals would never choose to pay the cost and switch culture. However, there is another issue at stake. Social interaction within the population is valuable, providing opportunities for economic exchange of varying types. Social ties (or 'links') allow for exchange of valuable information. For example, personal contacts play a large role in information about job opportunities and referrals, suggesting an important effect of social interaction on employment outcomes and wages (see Jackson, 2009, for a review of the literature). Social interaction can also provide other economic opportunities, including trade, favor exchange, or economic support such as risk sharing and other valuable joint endeavors (Angelucci et al., 2012). We assume all individuals provide the same opportunities for economic exchange.[20]

Crucially, personal connections and social interaction require commonality of some degree.

[19] Of course there may be multiple activities and behaviors associated with religion and multiple non-cultural activities. We extend the cultural and non-cultural activities that can be chosen by the population to multiple dimensions in Online Appendix B.

[20] There are two ways to relax this assumption in the current framework. The first is for one group to have greater opportunities, so that, all else equal, members of that group would make a more valuable partner. The other is to include some degree of 'love of diversity' to allow for benefits of getting different information and opportunities from different types. To introduce this one might assume that the first tie with someone of a different type is highly valuable, while the value may decline the more contacts of that type. The key trade-off of the model remains in place provided coordination remains important to interaction.

If two individuals do not speak the same language it limits exchange of information, discussion and agreement on trade, and any other activities that involve verbal communication (Lazear, 1999). Diversity more generally has been found to reduce communication and interaction within organizations (see Williams and O'Reilly, 1998, for a review). Communication difficulties aside, if two individuals take part in completely different activities then not only do they rarely meet (thus reducing opportunity for exchange), but even when they do meet they may not have relevant information to exchange.[21] There are two sides to this story: on the one hand a lack of commonality can make forming a tie more costly or difficult, and, on the other, it will make a tie less valuable if there are complementarities in information exchange and activities.

We model this formally as follows. Individual i chooses whether or not to form a social tie with the other $n-1$ individuals in the population. If individual i forms a social tie with individual j, we denote this by $g_{ij} = 1$, if not $g_{ij} = 0$. Player i's choice of social ties can be represented by a vector of 0's and 1's, $g_i = (g_{i1}, g_{i2}, \ldots, g_{in})$, where $g_{ii} = 0$. If i forms a social tie with j, the value of that social tie is increasing the more activities the two individuals have in common. The value i receives from a social tie with j is

$$\alpha\pi_1(x_i, x_j) + (1-\alpha)\pi_2(y_i, y_j) - L,$$

where

$$\pi_1(x_i, x_j) = \begin{cases} 1 & \text{if } x_i = x_j \\ 0 & \text{if } x_i \neq x_j \end{cases}$$

$$\pi_2(y_i, y_j) = \begin{cases} 1 & \text{if } y_i = y_j \\ 0 & \text{if } y_i \neq y_j \end{cases}$$

and $0 < L < 1$.

There is a fixed cost to forming a social tie, L. The benefit from a tie is increasing in commonality of actions. Note that the parameter α measures the relevance of cultural versus non-cultural actions in economic exchange. The framework can be interpreted in one of two ways. Either, there is a fixed value normalized to 1 of having a personal connection with another individual in the population, where the cost or difficulty of forming that tie depends on how much the two individuals have in common. Alternatively, we can think of a fixed cost L to forming a personal connection, where the possibility (or value) of economic exchange is

[21] For further discussion on the need for coordination in interaction see Lazear (1999) and Kuran and Sandholm (2008).

increasing the more individuals have in common.

2.3 Payoffs

Each individual $i \in \{1, .., n\}$ chooses a cultural action $x_i \in \{x^M, x^m\}$, a non-cultural action $y_i \in \{y^A, y^B\}$, and social ties $g_i = (g_{i1}, g_{i2}, \ldots, g_{in}) \in \{0,1\}^n$. An individual's strategy is thus represented by the vector

$$s_i = (x_i, y_i, g_i) \in S_i. \tag{1}$$

The utility of individual i of type $k \in \{M, m\}$ is given by

$$u_k(s_i, s_{-i}) = \sum_j (\alpha\pi_1(x_i, x_j) + (1-\alpha)\pi_2(x_i, x_j) - L)g_{ij} - c_k(x_i). \tag{2}$$

The model presented describes an n-player game where each player $i \in \{1, \ldots, n\}$ chooses a strategy $s_i \in S_i$ and receives a payoff $u_k(s_i, s_{-i})$. The strategy profile $(s_1^*, s_2^*, \ldots, s_n^*)$ is a Nash equilibrium if $u_k(s_i^*, s_{-i}^*) \geq u_k(s_i, s_{-i}^*)$ for all $i \in N$ and for all $s_i \in S_i$. We refer to a strategy profile $(s_1, \ldots, s_n)$ as a 'state'. We examine pure strategy equilibria.

The set-up is akin to a version of a battle-of-the-sexes game between groups. An individual gets a higher payoff the better he coordinates with his social ties, allowing for greater economic exchange. But, he would rather coordinate on his own cultural practices. The framework has the additional twist that each individual chooses his social ties. An individual also gets a higher payoff the more social ties he has with whom he is coordinating, again allowing for greater economic exchange. The assumption here, that more contacts are better, has support from the literature on social interaction (Currarini et al., 2009; Currarini and Vega-Redondo, 2011). It implies that individuals might trade-off cultural costs against the benefits of more contacts.[22]

Note two things. Individual i cannot differentiate his choice of action by social tie. Intuitively, this requires that an individual be 'consistent' in his behavior within the given population.[23] For example, it is often not possible to adopt two different religions. Religious activities might take place at the same time, the different practices might be contradictory, it could be too time-consuming to do both, or it might be made impossible because of hostility from others.

[22] We discuss alternative specifications of this assumption in Online Appendix B.

[23] A population consists of n individuals who each have the opportunity to interact with all the others. Thus a population refers to a workplace, a neighborhood, a school, etc. An individual may interact in multiple populations and play different strategies in different populations. This framework then does not prohibit different behaviors at work or school from those in the neighborhood, for example.

Second, social ties are one-sided. If i forms a social tie with j then the value of the social tie accrues to individual i but not to individual j unless individual j forms a social tie with i. In equilibrium, however, if i forms a social tie with j then j will also form a social tie with i. The model of link formation presented here is not the only way to model social interactions and is chosen to simplify the exposition (see Online Appendix B for further discussion).[24]

3 Analysis of the Single Generation Model

In this section we characterize the Nash equilibria of the game and provide comparative statics. We first introduce two assumptions made to simplify the exposition. Assumption 1 rules out the uninteresting case where a single individual prefers to maintain his cultural action even if the rest of the population all adopt a common, different cultural action.

Assumption 1 *(No Man is an Island)*

$$(1-L)(n-1)-c>\max\{0,(1-\alpha-L)(n-1)\}.$$

Second, we assume that when an individual is indifferent between forming a tie or not, the tie is formed.[25]

3.1 Characterization of Nash Equilibria

Proposition 1 describes the Nash equilibria of the game. Nash equilibria take one of three contrasting forms: (*i*) assimilation, (*ii*) segregation, or (*iii*) multiculturalism. Nash equilibria are characterized by thresholds on the share of immigrants in the population: if the immigrant group is small, assimilation states are the only equilibria; if the immigrant group forms a large enough share of a population, different social structures can emerge in equilibrium. For convenience we first introduce our formal definition of assimilation, segregation, and multiculturalism.

[24]The Nash equilibria in this paper all satisfy the definition of *pairwise stability*, an important measure of stability in network formation (Jackson and Wolinsky, 1996), although these are not the only pairwise stable outcomes. Alternatively, a two-sided link formation model, related to pairwise stability, where an individual could delete any number of links and form any number of agreed upon links would produce the same outcomes as the link formation model we use.

[25]Both can be relaxed.

Definition 1 *A state $s = (s_1, \ldots, s_n)$ is defined as:*

Assimilation *if all individuals adopt the same actions, $x_i = x_j$, $y_i = y_j$ $\forall i, j \in N$, and each individual forms a social tie with all other individuals.*

Segregation *if type M adopt action x^M, type m adopt action x^m, and each individual forms a social tie to all other individuals of the same type as him but does not form a social tie to individuals of a different type.*

Multiculturalism *if type M adopt action x^M, type m adopt action x^m, both types adopt the same non-cultural action $y_i = y_j$ $\forall i, j \in N$, and each individual forms a social tie to all other individuals.*

Proposition 1 *States which satisfy the definition of assimilation, segregation, and multiculturalism are the only possible Nash equilibria of the game. Further,*

(i) Any assimilation state is always a Nash equilibrium;

(ii) There exists a Nash equilibrium which satisfies the definition of segregation if and only if the share of the minority group in the population weakly exceeds δ;

(iii) There exists a Nash equilibrium which satisfies the definition of multiculturalism if and only if the share of the minority group in the population weakly exceeds η;

where

$$\delta = \max\left\{\frac{1}{2} - \frac{c+L-1}{2n(1-L)}, \frac{1-\alpha-L}{2(1-\alpha)-L} + \frac{1-\alpha}{(2(1-\alpha)-L)n}\right\}, \qquad \eta = \begin{cases} \frac{1}{2} - \frac{c-\alpha}{2n\alpha} & \text{if } 1-\alpha \geq L \\ 1 & \text{if } 1-\alpha < L \end{cases}$$

The proof is found in Appendix A. Only three types of social structure are Nash equilibria. Assimilation: one group pays the cost of switching cultural action to facilitate interaction with the other group. Segregation: groups do not pay the cost of switching cultural action but restrict interaction within their own group. And a third structure, multiculturalism, in which groups each maintain their respective cultural action but adopt a common non-cultural action. Crucially, coordination on these non-cultural practices enables interaction across groups.[26]

[26] These three distinct forms of social structure are evident in scholarly discussion of communities in the United States following mass migration (Gordon, 1964). Assimilation among immigrants in certain communities persuaded early scholars that a single 'American culture' would prevail. As it became clear that not all communities assimilated, but some 'retained distinctive economic, polical and cultural patterns long after arriving in the United States', scholars accepted that assimilation was not the only possibility for diverse communities (Bisin and Verdier, 2000). Even early on, however, there was discussion of a 'third way' where immigrants could become 'American' and integrate but also maintain some cultural distinction.

In our framework, an individual's payoff from taking an action is weakly increasing in the number of others that take that same action. It is important to highlight the role that such strategic complementarities play in our findings. First, this interaction between the individual's payoff and what the rest of the community are doing gives rise to multiple equilibria, allowing similar populations to end up in contrasting states. To see this, suppose individual i's group assimilates, so everyone in the population adopts the other group's cultural practices. Maintaining his own cultural practices then becomes very costly to i since it hampers interaction with everyone else, thus he can do no better than assimilate also. Hence assimilation is clearly an equilibrium. Now consider the other extreme: suppose everyone in i's group segregates and maintains their own cultural practices. Maintaining his own cultural practices is now much less costly to i, since he can still interact fruitfully with everyone in his group. Thus segregation may also be sustainable as a Nash equilibrium. Second, strategic complementarities create a multiplier effect, driving members of a group towards doing the same thing.[27] Even if we add some within group heterogeneity – see Online Appendix B for this addition to the model – because of the multiplier effect members of the same group will still have a tendency to do the same thing.

The two features discussed above lead to the key finding that equilibria are characterized by thresholds on the share of the minority group. When the minority group is small, assimilation states are the only equilibria; segregation and multiculturalism are equilibria only in populations where the share of the minority group is above the thresholds given in Proposition 1. An easy way to see this is to observe that maintaining a distinct culture is only 'worth it' if enough others do so as well. Segregation is an equilibrium when no individual in the minority group is willing to pay to switch culture and interact with the larger majority group, which is true only when the share of the minority group is large enough. Thus only if the share of the minority group reaches this 'critical mass' or 'threshold' can segregation be sustained. An analogous intuition holds for multiculturalism.[28] At these critical masses, a small change in the share of the minority group can result in a large change in equilibrium social structure. This is illustrated in Figures 1 and 2.

[27] By multiplier effect we mean the following. Once one individual adopts a particular action this raises the relative payoff to that action, which may then induce other agents to adopt the action, which further raises the payoff to that action, which may induce further agents to adopt, and so on.

[28] Under multiculturalism the minority group will interact with the majority group, but the benefit of interaction is lower than if they were to assimilate, and therefore the same intuition holds.

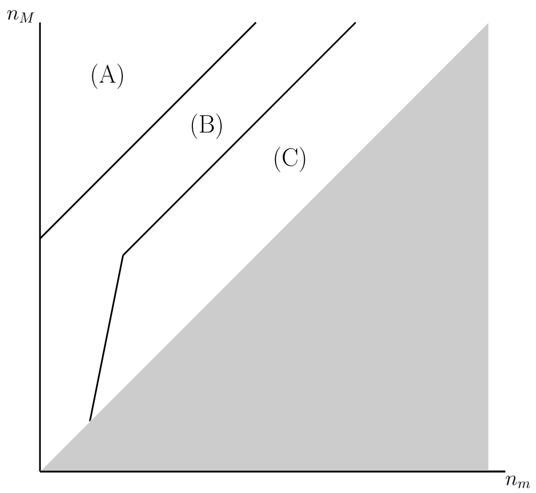

Figure 1

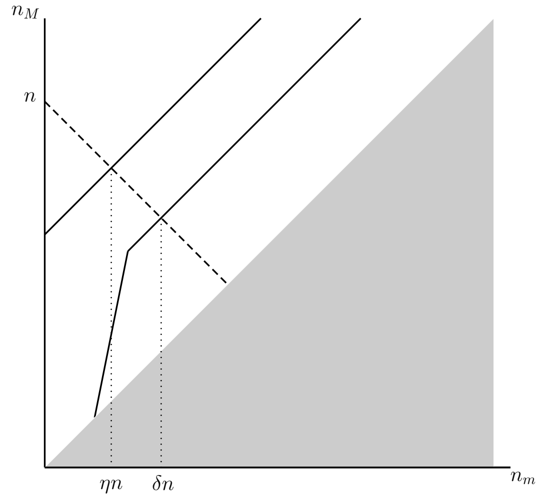

Figure 2

The Nash equilibria are illustrated for parameter values $1 - \alpha \geq L$. The axes measure the size of each group. Group M is only in the majority above the 45° line, so the area below the 45° line is greyed out. Assimilation is a Nash equilibrium in areas (A), (B), and (C) in Figure 1. Multiculturalism is an equilibrium in areas (B) and (C). Segregation is an equilibrium in area (C) only. Figure 2 illustrates the same graph highlighting the results for a population of fixed size n. The dashed line shows all possible shares for the minority group, from $n_m/n = 0$ to $n_m/n = 1/2$, for a population of a given size n. The dotted lines illustrate, for this population of size n, the size of the minority group above which multiculturalism and respectively segregation are equilibria. Note, the size of a minority group that can sustain multiculturalism is smaller than the size of the minority group that can sustain segregation because the cost to the minority group of multiculturalism is less than the cost of segregation.

3.2 Comparative Statics

The parameters c, α, and L shift the location of the thresholds described in Proposition 1. That is, c, α, and L determine the size of the critical mass of immigrants that is necessary to sustain segregation or multiculturalism. The parameter L is the cost of forming a link. Parameters c and α both measure 'culture' but have distinct interpretations. The magnitude of c is the cost of adopting the other group's cultural action. For example, it might be less costly to switch from speaking German to English, which is a closely related language, than it would be to switch from Italian. Contrast this with the parameter α, which measures the importance of cultural activities relative to non-cultural activities in everyday interaction and economic exchange. A high α environment is one in which cultural practices are frequently relevant to interaction. For example, a environment in which most social activities taking place are organised by churches and other religious organizations. In this example, a low α environment is one in which social activities related to religion comprise only a small part of daily life.

Corollary 1 *The threshold share of the minority group above which segregation states are Nash equilibria, δ, is decreasing in cultural distance, c; decreasing in the importance of culture, α;*

and decreasing in the cost of forming a link, L.

The threshold share of the minority group above which multiculturalism states are Nash equilibria, η, is decreasing in cultural distance, c; but increasing in the importance of culture, α; and increasing in the cost of forming a link L.

When cultural distance between groups, c, is higher, switching culture is more costly, and so smaller minority groups are more willing to maintain their own culture. This makes it easier to sustain both segregation and multiculturalism and the respective thresholds both fall.

Counterintuitively, an increase in the importance of culture, α, makes groups less willing to maintain their own culture under multiculturalism (the threshold rises). This is because, under multiculturalism, a higher α makes having a common culture more important to interaction, so maintaining one's own culture is more costly in terms of lost opportunities for exchange. Indeed, when culture 'dominates' everyday life (precisely $\alpha > 1 - L$), social interaction based on common non-cultural actions is not enough to sustain a tie and multiculturalism breaks down. To see this, consider again an environment where most social activities are related to religion. It is then difficult for individuals to maintain their own religious practices and integrate with those who adopt different practices. The set of equilibria may be reduced to just segregation and assimilation. An increase in α has the opposite effect on segregation making it a less costly option. This is illustrated in Figures 3 and 4.

A higher cost of forming a link, L, makes it easier to sustain segregation (lowers the threshold), by making assimilation a less attractive option for minority members. The costs of assimilation come from switching culture, while the benefits come from improved interaction with natives; a higher cost of link formation lowers the relative benefits of interaction with natives. In contrast, a higher cost of forming a link makes it harder to sustain multiculturalism. When the costs of forming a link are high, these can outweigh the benefits of interaction with the other group under multiculturalism.

To summarize, we find that one of only three structures – assimilation, segregation or multiculturalism – can arise in heterogeneous communities in equilibrium. Equilibria are characterized by a threshold, or 'critical mass'. When the minority group is small, assimilation is the only equilibrium. The size of the critical mass of immigrants necessary to sustain segregation or multiculturalism depends on cultural distance, the importance of culture in everyday life,

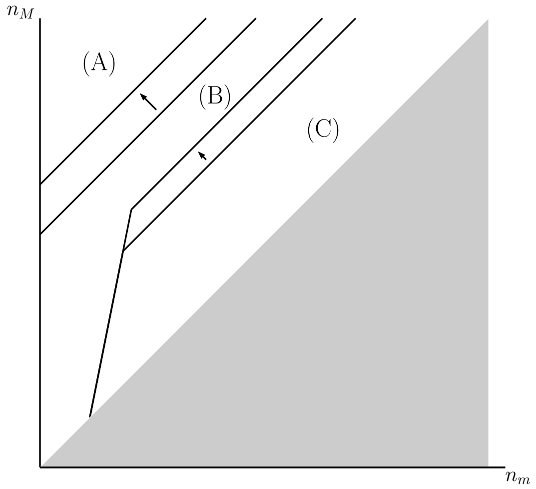

Figure 3: An increase in c.

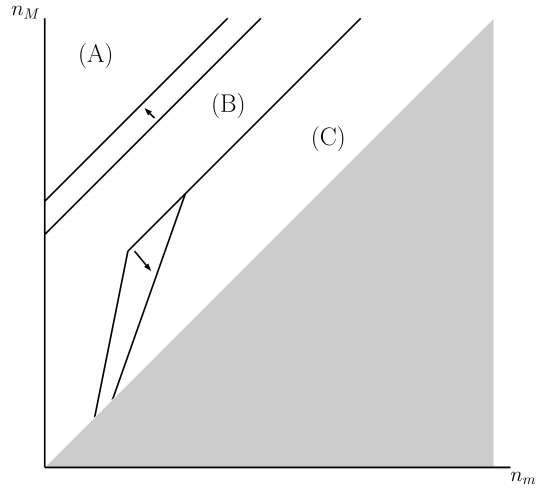

Figure 4: A reduction in α.

Figure 3 replicates Figure 1 and illustrates an increase in c. The arrows show that areas (B) and (C) expand with an increase in c. That is, the parameter ranges under which segregation and multiculturalism are equilibria expand with an increase in c. Figure 4 replicates Figure 1 and illustrates a reduction in α. The arrows show that area (B) expands and area (C) shrinks with a reduction in α. That is, the parameter range under which multiculturalism is an equilibrium expands with a reduction in α and the parameter range under which segregation is an equilibrium shrinks.

and the cost of link formation. In Section 5 we find evidence of such thresholds in heterogeneous communities in the United States in the age of mass migration: if immigrant groups hit a critical mass in the local community (the magnitude of which we estimate) they are much more likely to maintain their own practices and segregate.

A low importance of culture, α, appears at first to be a positive force for cross-group interaction since it makes it easier to sustain multiculturalism. However, our second comparative static result shows that a lower importance of culture leads to more polarizing behavior in segregation equilibria.

Corollary 2 *When $\alpha \leq 1 - L$ segregation is a Nash equilibrium when $n_m/n \geq \delta$ if and only if groups adopt different cultural actions* and *different non-cultural actions.*

When the importance of culture is low, $\alpha \leq 1 - L$, the segregation Nash equilibria require that the two groups adopt different cultural actions *and* different non-cultural actions. Groups differentiate their practices above and beyond *ex ante* cultural differences. This is not simply a result of coordination on different non-cultural actions. Instead the minority group must adopt a different non-cultural action y from the majority, in order to raise the cost of interaction with the majority group such that segregation can be sustained. This outcome is an equilibrium, despite the reduction in economic exchange that it entails. We refer to this type of segregation equilibrium as 'extreme segregation'.

Emergent Cultures

Non-cultural actions, that is activities and practices with no type-specific costs, play an important role in the social structures that emerge in equilibrium. It has long been recognised that when groups with distinct cultures interact, individuals do not simply choose between which of these cultures to practice. Instead, interaction can produce new 'emergent cultures', where actions that were previously culturally inconsequential take on an important role in uniting or dividing the community. This result can be seen clearly in the equilibria produced in our model.

In the multiculturalism equilibrium, groups maintain their different cultural actions, but the two groups adopt a common non-cultural action, either y_A or y_B. Construction of common practices is necessary to sustain integration when groups also retain some diversity of practices. This equilibrium can be related to the frequently advanced ideal that populations can maintain, for example, their diverse religions while at the same time having a common national identity and culture.[29] We show that this 'ideal' is feasible when culture is not 'all encompassing' in society; groups must be able to find enough common ground to be able to interact.

At the other end of the spectrum, under the extreme segregation equilibria, practices can become more polarized: groups differentiate their behaviors and practices beyond different cultural actions. Harris (2009) writes that some minority groups hold 'secondary cultural differences . . . that emerge after the two groups have been in continuous contact.' We show that when α is low, segregation can be maintained only when each group coordinates on distinct non-cultural actions. Rather than bridging the gap between groups, the non-cultural action is used to emphasise differences between them. For example, groups practising two different religions *ex ante* will also differentiate other behaviors that are un-related to religious requirements (for example, dress code, increased food restrictions, and even sports).[30] This appears to be a novel explanation of this type of behavior and we find it to be a positive signal of a model purporting to explain cross-cultural interaction that there is an equilibrium admitting this possibility.[31]

[29] As early as 1915, at a time of fervent discussion of integration of immigrants in the United States, Harvard philosopher Horance Kallen described the possibility of the United States being 'a democracy of nationalities, cooperating voluntarily and autonomously through common institutions' where 'the common language . . . would be English, but each nationality would have . . . its own peculiar dialect or speech, its own individual and inevitable esthetic and intellectual forms.'

[30] Berman (2000) describes the birth of Ultra-Orthodox Judaism in the late 18th and 19th Centuries which followed emancipation and the possibility of greater integration with the local European populations. The Ultra-Orthodox 'were not only conservative about rejecting new forms of consumption . . . but amplified existing restrictions', such as introducing new dietary restrictions, and 'changed existing customs (dress codes, speaking Yiddish) into religious acts'.

[31] A number of papers examine this type of outcome in detail including Berman (2000) on Ultra-Orthodox Judaism; Austen-Smith and Fryer (2005) on why some black students deride working hard at school as 'acting white' and act in opposition to this; and Akerlof and Kranton (2002) and Bisin et al. (2011) on the emergence of oppositional identities, whereby groups increase their

4 Multiple Generations with Cultural Transmission

Section 3 characterizes the equilibria for a single generation. In this section we incorporate the framework into a model of cultural transmission over generations. Considering multiple generations is important for two reasons. First, it may be that some outcomes that are Nash equilibria in a single generation are likely to die out in the long run. Second, since the static model has multiple equilibria for a given configuration of parameters, it is useful to consider whether some Nash equilibria are more 'likely' than others, thus the multiple generation framework acts as an equilibrium refinement.

We incorporate dynamic features into the single-generation model. First, we assume a multiple generation framework where parents pass on their type to their children (a simplified version of Bisin and Verdier, 2000). Second, we do not assume that the population automatically starts off at an equilibrium. Children, once born, choose their actions and social ties optimally as above, but taking as given the existing choices of the population they are born into. This means that all else equal, a child born into a more assimilated group may behave differently to one born into a more segregated group, again capturing the importance of the behavior of the rest of the community. Finally, we allow for a few members of a new generation to 'experiment' or 'make mistakes'. We can also think of these as temporary shocks which happen infrequently. For example, suppose the population is in a state where members of the minority group maintain their minority culture. In the next generation, however, a few children in the minority group 'experiment' and try out majority culture. This can set in motion a move to a different Nash equilibrium: if enough people experiment it may be optimal for future minority group children to also adopt majority culture. These shocks allow populations to move away from 'less stable' Nash equilibria and towards 'more stable' ones.[32]

The dynamic is modeled formally as follows. Suppose at (discrete) time t the population of size n is in some state which is described by $(s_1^t, s_2^t, \ldots, s_n^t)$, where s_i^t is the choice of actions and social ties of individual i at time t. A strategy at time t, s_i^t, is given by (1), as in the single generation case. The state need not be a Nash equilibrium. At time $t+1$ there is an independent probability $q \in (0,1)$ for each individual i (parent) that they die and are replaced by a child, also denoted i. The child is the same type as the parent, M or m, and chooses an

identification with their own culture in order to reduce the psychological cost of interacting with those who adopt a different culture.

[32] As described in Young (1993), the process we model 'selects the equilibrium that is easiest to flow into from all other states combined, including both equilibrium and non-equilibrium states'.

action which maximizes his utility given the state in period t:

$$s_i^{t+1} \in \arg\max_{S_i} u_k(s_i^{t+1}, s_{-i}^t)$$

where $u_k(s_i^{t+1}, s_{-i}^t)$ is given by (2). Under this dynamic the population moves to a Nash equilibrium, and once it reaches an equilibrium it will stay there for all future generations. However, we also allow that with probability ϵ the child instead adopts some other strategy randomly. This is how we model experimentation by children.

We examine the case where the probability of experimenting, ϵ, is small. The process detailed above defines an aperiodic and irreducible Markov chain. The following proposition tells us that, for any given parameter values, a single outcome will be observed almost all of the time.[33] Which outcome is observed in the long-run depends on the parameters of the population in a similar way to the single-generation case. We make the following assumption, which ensures cultural distance is sufficiently important that the trade-off between maintaining one's own cultural practices and forming social ties is economically relevant.[34]

Assumption 2 $c \geq (1 - L)(n_m + 1)$

Under Assumptions 1 and 2 we get the following result.

Proposition 2 *When the probability of experimenting is small, $\epsilon \to 0$, and $t \to \infty$, the following outcome is observed with probability tending to* 1*:*

(i) When $1 - \alpha < L$ there exists a threshold δ^* such that

Assimilation by the minority group is observed if $n_m/n \leq \delta^* - 1/n$;

Segregation is observed if $n_m/n \geq \delta^* + 1/n$.

(ii) When $1 - \alpha \geq L + (\frac{1}{n_M - 1})$ there exists a threshold η^* such that

Assimilation by the minority group is observed if $n_m/n \leq \eta^* - 1/n$;

Multiculturalism is observed if $n_m/n \geq \eta^* + 1/n$.

In the long-run, a unique outcome emerges (or is more likely to emerge). Proposition 2 tells us that if the minority group is small, assimilation occurs and it is the minority group that

[33] There is a unique stationary distribution which puts all weight on a single state.

[34] It is made for ease of exposition. If Assumption 2 does not hold, then the cost of switching culture is low, and both assimilation to M or assimilation to m could persist in the long run. See the proof of Proposition 2 for details.

assimilates. It also shows that multiculturalism can be sustained as a stable long-run outcome in a diverse population. This occurs when culture is not too important in everyday life (α low) and the minority group is large. If culture is very important (α high) and the minority group is large, then we will see long-run segregation of groups.

Analogous to the single-generation model, higher cultural costs, c, allow populations with a smaller proportion of minority individuals to sustain segregation or multiculturalism in the long run. A lower importance of culture, α, can ensure multiculturalism is a long-run outcome instead of segregation, and allows multiculturalism to be sustained in populations with a smaller proportion of minority individuals. The result that different cultural practices can persist, under certain parameters, even in the long-run, is consistent with the findings and discussion in Bisin and Verdier (2000).

The proof of Proposition 2 involves assessing the minimum number of shocks needed to induce a transition from one Nash equilibrium to another, and then aggregating to determine which of the equilibria is most likely to occur in the long-run. We simplify this by using a 'tree pruning' argument, the details of which can be found in the proof in Appendix A. The $+1/n$ and $-1/n$ terms added to the thresholds and the small positive term $+(\frac{1}{n_M-1})$ in Proposition 2 are there to avoid having to make a more complex statement about what happens at the boundary parameters between one long-run outcome and another. Details and precise values for thresholds can be found in Appendix A and Online Appendix B.

The forces driving the results of Proposition 2 are intuitive. First, why does the minority group assimilate and not the majority group? Suppose in generation t the minority group and majority group adopt their respective cultural practices. In generation $t+1$ some members of the minority group experiment by adopting majority culture and interacting with the majority group. If enough members experiment, then future minority group members will choose to switch and also adopt majority culture. Since the minority group is smaller, less experimentation is required to induce further minority group members to optimally choose to assimilate to majority culture than if we repeat the same scenario with the majority group. For a similar reason, assimilation occurs only when minority individuals constitute a small share of the population: the smaller the minority group, the less experimentation it takes to induce movement in the minority population towards assimilation.

A natural extension of the multiple generation framework would allow the costs of practices to change over generations. If a parent assimilates, the cost to the child of switching culture may be less than the cost the parent faced. Alternatively, where a parental generation segregates and further differentiates non-cultural actions, the child may face type-specific costs over the actions for which the parents faced no cost.[35] This would be an interesting avenue for future research.[36]

5 Evidence from the Age of Mass Migration

The 'Age of Mass Migration' in the late 19th and early 20th centuries is one of the most important episodes of migration of the modern era. From 1830 to 1930, 38 million people arrived in the United States. They joined a population that in 1830 consisted of only 13 million people, and which Alba (1985) describes as having both 'culture and institutions, [that] derived largely from the English models'. Immigrants came to America from around the world, including virtually every country in Europe.[37]

We look at heterogeneous communities throughout the United States in this era and ask how this heterogeneity manifested itself. Did immigrants adopt the behaviors and practices of the local population? Did they integrate or form segregated communities? The setting provides a natural environment in which to examine some of the implications of our model. Importantly, it provides us with a large number of heterogeneous communities with different immigrant groups of varying sizes.

We test two key predictions of our model: that group behavior changes sharply once the share of the immigrant group in the community reaches some (*ex ante* unknown) critical threshold, and that the location of this threshold varies in the predicted way with the cultural distance between immigrants and natives. In particular, we consider two choices facing immigrants: speaking English, and in-marriage (homogamy).[38]

Testing these predictions of our model is important for two reasons. First, if our framework captures an important trade-off, then we should expect to observe such thresholds in data.

[35] Observe, it is not the case that cultural costs necessarily fall over generations. Suppose one individual from the parent generation experiments and assimilates while the rest of his group segregates. Then the cost to his child of speaking the other group's language falls somewhat. However that child may still find it preferable to choose to segregate, in which case the costs of speaking the other group's language may go up again for the grandchild.

[36] One way to do this would be to combine the current framework with features of Kuran and Sandholm (2008), which allows costs of taking an action to change as actions taken change.

[37] Not all immigrants remained in the US permanently (Bandiera et al., 2013). We will discuss later the implications of this for our results.

[38] In contrast to our baseline modeling assumption, immigrants could continue to speak both their native language and learn English. As mentioned previously, our results will hold provided that the relative benefit of learning English increases the greater the number of members of the immigrant group who learn it.

Second, evidence of such thresholds, and hence of the forces which drive our model, has many implications for policy.

5.1 Data Description

Data Sources

We use data from the 1900, 1910, 1920, and 1930 census samples provided by IPUMS. These are 5% (1900 and 1930) or 1% (1910 and 1920) samples. These censuses asked individuals whether or not they speak English. They also allow us to link (almost all) married individuals to their spouses, so we can measure whether individuals marry endogamously (i.e. within the same nationality).[39] Important to our question, in these particular years individuals were asked their country of origin, and – unlike earlier and later censuses – their year of arrival in the US, allowing us to control for how long individuals have lived in the US. Focusing on adult household heads, we have a repeated cross-section with 611,000 migrants.

The census samples provide information on an individual's place of residence down to the county level. Counties are small administrative units, with on average 63 counties per state, and an average population of 5300 households per county in 1900, of which 1300 were headed by immigrants.[40] We also use data from Spolaore and Wacziarg (2009), collected by Dyen et al. (1992) and Fearon (2003), on the linguistic distance between pairs of languages.

Data Construction

We treat counties as the relevant population for our analysis. That is, individuals are presumed to make their decisions about what behaviors to adopt and with whom to form ties, taking those in the county as the pool of people they could potentially interact with.[41]

Within each county, we define an immigrant group as consisting of all household heads of the same nationality (country of origin), with the exception that we group Germany, Austria, and Switzerland into a single group; Sweden, Denmark, and Norway into a single group; and

[39] Individuals whose spouses do not live in the same household cannot be matched. They make up less than 3.1% of married individuals in our sample.

[40] Note, the difference between these figures and Table 1 comes both from changes over time in county size (the table pools all censuses), and from the fact that larger counties have more observed cohorts, so receive more weight in the table.

[41] Counties are the finest population partition available to us. It seems possible that decisions are made based on a more local population of people, such as the town/village. In that case we observe only a noisy measure of the population share and population decisions, likely attenuating our estimated effect and reducing power.

the Netherlands and Belgium into a single group.[42, 43] Our qualitative results are unchanged in the absence of this aggregation. Henceforth, by 'nationality' we refer to these groups. In each county, the 'nationality share' is the proportion of sampled adult household heads in that county who have that nationality: this is the empirical analogue of n_m/n in our model.[44]

Since our model predictions are at the group level, we aggregate the behavior of individuals into 'cohorts'. Cohorts are defined by nationality, d; year of arrival (in 10 year bands), a; time since immigration (also grouped, to the nearest 10 years), t; and county, o.[45] Splitting the observed behavior of immigrant groups into these cohorts allows us to control for arrival year and tenure effects that might be important in explaining behavior and might vary systematically by nationality. For example, both ability to speak English and out-marriage are likely to be positively related to tenure in the United states. Additionally, varying conditions and policies over the decades we study, such as use of schooling to influence assimilation (Bandiera et al., 2015), may have affected these decisions.

We measure English acquisition by the proportion of a cohort that reported to the enumerator that they spoke English.[46] For this analysis we exclude British, Irish, and Canadian immigrants, since their mother tongue is likely to be English anyway. To proxy for social segregation, we construct a measure of the extent to which each cohort marries people of the same nationality. We calculate the proportion of married members of the cohort that are married to someone of the same nationality.[47] Although marriage is a partial description of social connec-

[42] We focus on household heads since they were less likely to have received education in the US which would change their decision-making. At the time they also were likely to have made choices for the whole household. In the case of in-marriage it also avoids the double counting of homogamous relationships.

[43] We made these groupings due to use of a common language and because accounts of communities in the US suggest these amalgamations are appropriate. Austrians spoke, in main part, a form of German, and the Swiss who emigrated were largely German speaking. At the time we are considering, Danish remained the official language of Norway, and also Swedish, Danish, and Norwegian are considered mutually intelligible. The Belgians who emigrated to the US were largely of Flemish descent, and so spoke Dutch.

[44] Note that this treats all individuals who are not members of that nationality group as though they were members of the majority group. In all counties, the largest other group that immigrants could consider joining is the native group. Hence, if an immigrant group is considering switching to another culture, this is likely to be the most profitable. Although in some cases it may be lower cost to join another immigrant group due to their lower cultural distance, this is likely mitigated both by our amalgamation of individuals from certain countries and the fact that these groups are typically much smaller than the natives. In our analysis sample there are only 33 counties where more than one group exceeds 10% of the county population, 4 counties where more than one group exceeds 15%, and none at a threshold of 20%. An alternative empirical specification would be to define the denominator for nationality share as the sum of the number of immigrants of that nationality and natives, thus excluding other immigrants. Our results are robust to this redefinition.

[45] In particular, arrival years are grouped as 1886-1895, 1896-1905, and so on. Tenure (time since immigration) is then measured as the difference between the midpoint of the arrival year band (1890, 1900, etc) and the census year.

[46] Enumerators were instructed 'Where difficulty is encountered in making the head of the family understand what is wanted, you should call upon some other member of the family who is able to speak English . . . if no member of the family can aid you in your work, then the assistance of some neighbor of the same nationality and able to speak English should be obtained, whenever possible.' Where this was also not possible, paid interpreters were used. Hence this question is likely to have been able to meaningfully capture whether individuals were able to speak English. https://www.census.gov/history/pdf/1900instructions.pdf

[47] The Coleman index provides an alternative popular measure of segregation (see Currarini et al., 2009). It is not suitable for our context as it requires knowledge of the potential marriage market for a particular cohort at the time marriage decisions were taken, something we do not observe. Our preferred measure is therefore the simple ratio of in-marriage to total marriage for a cohort, which has the additional benefit of being easy to interpret.

tions, it is clearly defined and measured for this period, and marriage decisions are considered to be strongly revealing of the dimensions that divide a society (for a review see McPherson et al., 2001).

Table 1 shows the mean and variance for these outcomes, as well as nationality size (n_m in our model), county size (n), and nationality share (n_m/n), across the analysis sample. We also provide the within-nationality variance, and the share of total variance that is within variance. From this it can be seen that there is significant within nationality variation that we can exploit in our analysis, allowing us to avoid concerns that the effects we capture come only from differences across nationalities.

To measure the cost of learning English, c in our model, we use the data on linguistic distance. The data from Dyen et al. (1992) define linguistic similarity between a pair of languages based on the proportion of frequently used words that share a common root.[48] We assume that immigrants' mother tongue is the majority language spoken in their country of origin. We also assume that the cost of switching language to English is greater for those immigrants whose mother tongue is more dissimilar to English. We then split cohorts into 'near' or 'far' from English, defining a language as far from English if less that 40% of the words are mutually intelligible with English.[49]

For our analysis we also only consider cohorts with at least 30 observations, so that the average group behavior is well estimated. This means that we exclude counties where the number of immigrants was very small. There is a trade-off between choosing a low minimum threshold for immigrant group size, to minimise the bias that comes from effectively ignoring very small community shares, and choosing a high threshold to reduce noise in estimated group behavior. We test the sensitivity of our results to this by also using cut off points of 20 or of 40, and find no qualitative difference. As an alternative sensitivity test, we also performed our analysis with weighted regressions. In particular, the weight given to an observation was equal to the number of observations with the same nationality share in the full sample, divided by the number with that nationality share in the estimation sample.[50] This over-weights observations with small nationality shares, since these are the ones excluded by our group size thresholds.

[48]This measure of distance is only available for Indo-European languages. We impute some of the missing data using a variable from Fearon (2003) which measures linguistic proximity using a 'language family tree'. Our results are qualitatively robust to instead dropping these observations with missing data.

[49]We take this cut-off from Advani and Rasul (2015), who choose it because there are no languages in the range $30-50\%$ mutually intelligible with English, so it forms a natural break. According to this metric, Spanish and Greek are considered far from English, whilst German and Danish are considered close.

[50]For constructing weights we split nationality share into deciles, with all observations in the decile being given equal weight.

Table 1: Descriptive statistics for key variables

	(1) Mean	(2) Total Variance	(3) Within Var	(4) Within/Total
Share speaking English	.901	.023	.011	.489
Share of in-marriages	.609	.055	.022	.391
Nationality Size ('000s)	.459	.391	.374	.957
County Size ('000s)	8.79	106.9	96.5	.903
Nationality Share	.085	.007	.005	.732
Observations	1746	1746	1746	1746

Notes. Observations are at the cohort level. Cohorts are defined by nationality, county, tenure in the US grouped to the nearest 10 years, and year of arrival in 10 year bands. We include only cohorts with at least 30 individuals, to ensure that cohort averages are well measured. Share speaking English measures the proportion of a cohort that reports speaking English. Share of in-marriages measures the proportion of married individuals in a cohort who are married to someone of the same nationality. Nationality size measures the number of people, in thousands, of the same nationality (potentially of different cohorts) living in the same county. County size measures the number of people, in thousands, living in the same county. Nationality share is the ratio of nationality size to county size. Columns (1) and (2) provide the mean and variance for these variables. Column (3) shows the remaining variance after removing nationality fixed effects, and Column (4) provides the share of total variance that remains once nationality fixed effects are removed.

Again we find similar results. These results are presented in Online Appendix C.

In total we have 1746 cohorts, when we impose a minimum cohort size of 30, of which Germans and Italians are the biggest groups. Table C1 in Online Appendix C shows the number of immigrant cohorts we observe, and splits out some of the larger nationalities. It also shows how this varies depending on the minimum size we require for a cohort to be included in our sample.

5.2 Empirical Framework

To formally test for the existence of a threshold in community behavior we use a linear regression of the form

$$Y_{dato} = \beta_0 + \beta_1 \mathbb{1}\{z_{do} > \tau\} + \gamma_0 z_{do} + \gamma_1 z_{do} \mathbb{1}\{z_{do} > \tau\} + \kappa_d + \lambda_a + \nu_t + \varepsilon_{dato} \tag{3}$$

where Y_{dato} is the share of immigrants of nationality d, arriving in year a, with tenure t, in county o, who speak English (respectively, in-marry); z_{do} is the nationality share of immigrants of nationality d in county o; and τ is the proposed threshold immigrant share. κ, λ, and ν are respectively the nationality, arrival year, and tenure fixed effects. The coefficient of interest is β_1: the coefficient on the indicator function showing whether there is a 'break' in the level of speaking English (respectively, in-marriage) when the nationality share z_{do} crosses the threshold τ.

Since we do not know theoretically where the threshold τ should be, we use an iterative regression procedure, and test for significance using a Quandt Likelihood Ratio (QLR) test (Quandt, 1960; Andrews, 1993). We perform a sequence of regressions, testing a prespecified range of values for τ. For each regression we calculate the F-statistic, comparing the model with the threshold to the same model but without a threshold. We then select from among these regressions, the one with the highest F-statistic. The corresponding break point in that regression is then taken as the estimated location of the threshold, denoted τ^*. To test whether this threshold value is 'significant', we compare the F-statistic to the limiting distribution for this statistic under the null (Andrews, 1993), thus correcting for the multiple testing.[51, 52]

We also estimate Equation 3 separately for cohorts that are 'far from' and 'near to' English, and compare the point estimates, to see whether $\tau^*_{far} < \tau^*_{near}$, for both speaking English and in-marriage. However, we are unaware of any formal way to test for the statistical significance of any difference in location.

5.3 Results

Threshold in proportion speaking English

Figure 5 shows graphically the evidence for a threshold in the share of the immigrant group that speaks English, as its share of the community rises. Each dot shows the mean proportion that report speaking English for a two percentage point range of nationality share. That is, the leftmost dot shows the mean proportion of immigrants speaking English for all immigrants in groups that constitute 0-2% of their community. We also plot a kernel-weighted local mean, fitting it separately either side of a nationality share, z_{do}, of .31 (below we describe how this point was chosen). From this figure we can immediately see three things: (*i*) when immigrants make up only a small share of the population, the proportion of the cohort that speaks English is high, at around 90%; (*ii*) there is a sharp drop in this mean, to less than 40%, when the immigrant group reaches approximately 1/3 of the population; (*iii*) there is more variation in the mean after the threshold. These three findings are consistent with our model. The first result shows clear evidence that small groups of migrants are assimilated into American culture, almost all learning the language. The visually striking threshold is clearly suggestive of the type

[51] For robustness, we also use an entirely non-parametric approach to finding the threshold location, as proposed by Henderson et al. (2015). The intuition of this approach is similar to the parametric approach, searching over a sequence of thresholds, but without any functional form restrictions away from the threshold. The location of the thresholds we find are the same as for the parametric approach.

[52] An alternative method, followed by Card et al. (2008), is to split the sample into a test sample, which one can use to select between multiple potential thresholds, and an analysis sample where the selected threshold can be used. This allows conventional testing approaches with more standard distributions, but at the cost of reduced test power.

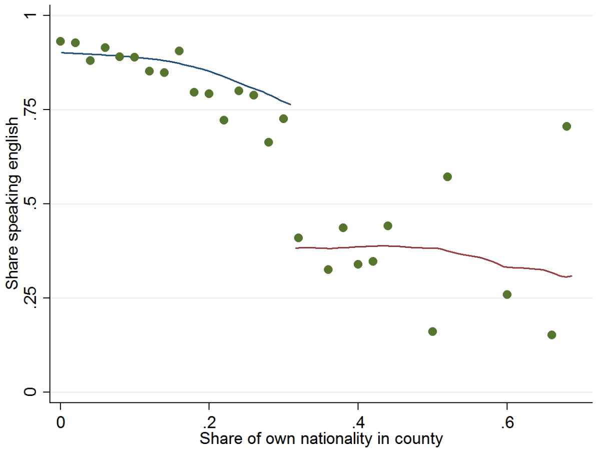

Figure 5: Share of people in the cohort that speak English

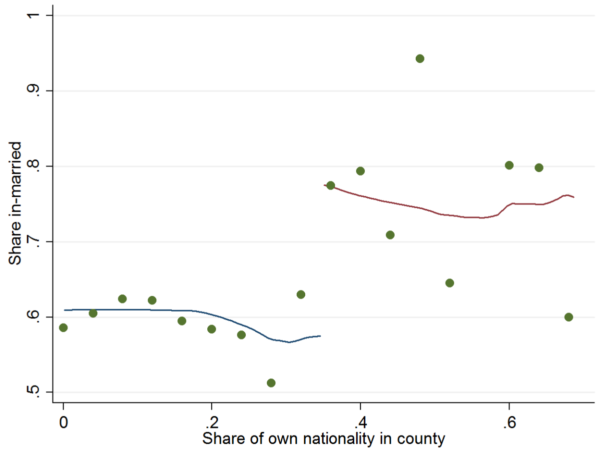

Figure 6: Share of married people in the cohort that are married to someone of the same nationality

Notes. The figures show the relationship between the share of a cohort that speak English (Fig 5) or marries within its nationality (Fig 6), and the proportion of adult household heads in their county that are of the same nationality. Dots show the mean share speaking English/in-married, grouping nationality share into bins two percentage points wide. The line shows the kernel-weighted local mean, estimated using an Epanechnikov kernel with bandwidth of 7% fit through the whole (i.e. unbinned) data. In each figure the local mean is estimated either side of an estimated threshold: .31 for Fig 5 and .35 for Fig 6. These points are chosen using the Quandt Likelihood Ratio procedure descibed in Section 5.2. Cohorts are defined by nationality, 10 year grouped arrival year, 10 year grouped tenure in the US, and county of residence. A minimum cohort size of 30 is used.

of strategic complementarity notion at the heart of our model. And the final result is consistent with the existence of multiple equilibria above the threshold, so that in some communities we may see assimilation outcomes even at these high nationality shares.

The choice of .31 for the break in our local mean plot comes from performing a formal test for the presence and location of a significant break. We use the QLR test described above, searching a grid between [.20, .40] with increments of .01, and test for the most likely value of the threshold, τ, in a sequence of increasingly flexible models. We consistently find a significant break in English acquisition, and find the most likely value for the threshold is when the immigrant group constitutes 31% of the population. Table 2 shows the results of the tests.

The first specification includes only a constant and a threshold term, so that any systematic variation in the share of the cohort speaking English can only be picked up as some kind of threshold effect. This amounts to imposing that the slope terms are zero ($\gamma_0 = \gamma_1 = 0$) and ruling out nationality, year of arrival, and tenure fixed effects ($\kappa_d = \lambda_a = \nu_t = 0$) in Equation 3. We find a strong support for the presence of a threshold, with an F-statistic of 423 (compared with a 1% critical value of 6.6, taken from Andrews, 1993), and the most likely location for the threshold is $\tau^* = .31$.[53]

[53] These results are robust to minimum cohort size chosen. Table C2 replicates Column (1) using different sample size thresholds, and also using the weighting method discussed earlier. The location and significance of the threshold τ^* remains unchanged. The same threshold is also found when we use the completely non-parametric search procedure suggested by Henderson et al. (2015).

Table 2: Testing for a threshold effect in share speaking English

Dependent Variable: Proportion of people in the cohort that speak English
Standard Errors in Parentheses
Cohorts defined by Nationality, County, Grouped Year of Arrival, Grouped Tenure

	(1) Unconditional	(2) With Fixed Effects	(3) With Slopes
Optimal threshold ($\beta_1 \| \tau = \tau^*$)	-.509***	-.257***	-.375***
	(.025)	(.017)	(.062)
Constant (β_0)	.885***	.878***	.913***
	(.004)	(.002)	(.004)
Optimal Threshold Level (τ^*)	.31	.31	.31
F-statistic	423	241	37
1% critical value for F-statistic	6.6	6.6	6.6
Cohort Fixed Effects	No	Yes	Yes
Slopes in Nationality Share	No	No	Yes
Observations	1272	1272	1272

Notes. *** denotes significance at 0.1%, ** at 1%, and * at 5% level, when treated as a standalone regression. The outcome measures the proportion of the cohort that speaks English. Cohorts with English, Canadian, and Irish nationalities are excluded from the sample, since English is likely to be their mother tongue. Minimum cohort size of 30. Cohort fixed effects are composed of nationality, arrival year (grouped), and tenure (grouped) fixed effects. Column (1) is a regression of share speaking English on a constant, and a dummy for whether nationality share in the county exceeds a threshold. Column (2) allows for cohort fixed effects. Column (3) allows the share speaking English to also have a slope in nationality share, and allows this slope to vary either side of the threshold. All covariates except the threshold are demeaned, so the constant (β_0) can be interpreted as an estimate of the mean share speaking English at nationality shares below the threshold. These specifications are run sequentially at different values of the threshold, varying the threshold between .20 and .40, at intervals of .01. We provide the results for the threshold among these which produced a regression with the highest F-statistic when tested against the null of no threshold. The value for this threshold, the estimated F-statistic, and the 1% critical value for this statistic (which corrects for the repeated testing, see Andrews, 1993) are provided at the bottom of the table.

Our next specification allows for nationality, arrival year, and tenure fixed effects, so we only impose $\gamma_0 = \gamma_1 = 0$. This allows for the possibility that, for example, Germans might live in groups with systematically smaller nationality shares than Italians and also find it easier to learn English, although even this sort of story is unlikely to give rise to so clear a threshold as seen in Figure 5. Similarly we now allow for variation in settlement patterns and language acquisition by groups arriving in different years, and who have been present in the US for different tenures. Again we can strongly reject that no threshold exists (F-statistic of 241), with the same location. We also now see that the point estimate for $\beta_1|\tau = \tau^*$ is reduced, showing that some of the reduction in English speaking is explained by the fixed effects.

Finally, in Column (3) we allow also for slopes either side of the potential threshold. Although our model as written does not have such effects, simple extensions of the model, such as allowing some heterogeneity in costs or benefits, might allow some sort of slope either side of the threshold. Even allowing for such effects, we find continued support for a threshold, again when immigrants make up 31% of the local population, with an F-statistic of 37.

Table 3: Testing for a threshold effect on share in-married

Dependent Variable: Proportion of the married people in the cohort that are married to someone of the same nationality
Standard Errors in Parentheses
Cohorts defined by Nationality, County, Grouped Year of Arrival, Grouped Tenure

	(1) Unconditional	(2) With Fixed Effects	(3) With Slopes
Optimal threshold ($\beta_1 \vert \tau = \tau^*$)	.151***	.065***	.166***
	(.042)	(.017)	(.049)
Constant (β_0)	.606***	.606***	.562***
	(.006)	(.003)	(.006)
Optimal Threshold Level (τ^*)	.35	.23	.23
F-statistic	13.0	15.0	11.5
1% critical value for F-statistic	6.6	6.6	6.6
Cohort Fixed Effects	No	Yes	Yes
Slopes in Nationality Share	No	No	Yes
Observations	1746	1746	1746

Notes. *** denotes significance at 0.1%, ** at 1%, and * at 5% level, when treated as a standalone regression. The outcome measures the married proportion of the cohort that is 'in-married' *i.e.* married to someone of the same nationality. Minimum cohort size of 30. Cohort fixed effects are composed of nationality, arrival year (grouped), and tenure (grouped) fixed effects.
Column (1) is a regression of share in-married on a constant, and a dummy for whether nationality share in the county exceeds a threshold. Column (2) allows for cohort fixed effects. Column (3) allows the share in-married to also have a slope in nationality share, and allows this slope to vary either side of the threshold. All covariates except the threshold are demeaned, so the constant (β_0) can be interpreted as an estimate of the mean share in-married at nationality shares below the threshold. These specifications are run sequentially at different values of the threshold, varying the threshold between .20 and .40, at intervals of .01. We provide the results for the threshold among these which produced a regression with the highest F-statistic when tested against the null of no threshold. The value for this threshold, the estimated F-statistic, and the 1% critical value for this statistic (which corrects for the repeated testing, see Andrews, 1993) are provided at the bottom of the table.

Threshold in proportion in-married

We now repeat the above analysis using proportion in-married as the outcome variable. Figure 6 is constructed in the same way as Figure 5, and tells a similar story to that seen with speaking English. We see that: (*i*) when immigrants make up only a small share of the population, the assimilation type outcome is apparent, with relatively less in-marriage; (*ii*) in-marriage rises sharply after the threshold; and (*iii*) we see increased variation after the threshold. The picture is less sharp here, since the choice of whether to in-marry only captures a single, limited dimension of interaction between different groups. Additionally, since the timing of the marriages are unknown, some of the in-marriage might reflect relationships formed prior to immigration, giving a relatively high level of in-marriage even when nationality shares are very low. Despite these limitations, at the threshold we see a jump of one-fifth, from less than 60% to more than 75% in-married.

Table 3 shows our main formal results on in-marriage, again testing for the presence and location of a significant break using the QLR procedure described above (searching a grid between [.20, .40] with increments of .01). Column (1) estimates our empirical specification without fixed effects ($\kappa_d = \lambda_a = \nu_t = 0$) or slopes ($\gamma_0 = \gamma_1 = 0$). We find a significant break (F-statistic of 13.0 against a 1% critical value of 6.6) in in-marriage when the immigrant group constitutes 35% of the population. Levels of in-marriage rise from 60% to 75% at this threshold.[54] Column (2) includes fixed effects, and Column (3) additionally allows slopes to vary. We continue to find a significant break in in-marriage (F-statistics of 15.0 and 11.5 respectively), although it now occurs earlier, when the immigrant group constitutes 23% of the population.

In summary, we show evidence of stable levels of in-marriage in communities where the immigrant group is below approximately a quarter of the population (once we account for fixed effects). When the immigrant group reaches this nationality share we find a significant jump upwards in rates of in-marriage. This discontinuous increase in segregation, even when using a partial measure of interaction, provides strongly supportive evidence of the mechanisms driving our model.

Higher thresholds when culturally closer

Our model predicts not only the presence of a threshold, but also that the threshold should vary with c. In Table 4 we now estimate the location and significance of the threshold separately for groups who are from countries with a language 'near to' or 'far from' English. Splitting the sample into these two cases, we estimate the unrestricted version of Equation 3, analagous to the third columns of Tables 2 and 3.

For speaking English (Panel A) we find a significant threshold at the 1% level for both far and near nationalities. As predicted, for near nationalities the threshold occurs at a larger nationality share ($\tau^* = .39$) than for far nationalities ($\tau^* = .31$). This is consistent with our theoretical finding that nationalities that are culturally closer to English need to make up a relatively larger proportion of the local population before they choose to retain their own language.

For in-marriage (Panel B), we see a significant threshold still for the near group, at 23% (significant at 5% level), but we do not find a significant threshold for the far group. In part this is likely due again to our measure of interactions being only a partial one.

[54] These results are robust to minimum cohort size chosen. Table C3 in Online Appendix C replicates Column (1) using different sample size thresholds, and also using a weighting method, and significance of the threshold τ^* remains unchanged. The same threshold is also found when we use the completely non-parametric search procedure suggested by Henderson et al. (2015).

Table 4: Comparing the Threshold Locations for Linguistically Far and Near Cohorts

Dependent Variable: (A) Proportion of people in the cohort that speak English; (B) Proportion of cohort in-married
Standard Errors in Parentheses
Cohorts defined by Nationality, County, Grouped Year of Arrival, Grouped Tenure

	(A) Speaking English		(B) In-Married	
	(1) Ling. Far	(2) Ling. Near	(1) Ling. Far	(2) Ling. Near
Optimal threshold ($\beta_1 \vert \tau = \tau^*$)	-.118***	-.115**	-.031	.182*
	(.029)	(.035)	(.034)	(.079)
Constant (β_0)	.876***	.977***	.761***	.402***
	(.005)	(.007)	(.006)	(.011)
Optimal Threshold Level (τ^*)	.31	.39	.39	.23
F-statistic	17.0	10.7	.80	5.3
5% critical value for F-statistic	3.8	3.8	3.8	3.8
1% critical value for F-statistic	6.6	6.6	6.6	6.6
Cohort Fixed Effects	Yes	Yes	Yes	Yes
Slopes in Nationality Share	Yes	Yes	Yes	Yes
Observations	810	462	810	936

Notes. *** denotes significance at 0.1%, ** at 1%, and * at 5% level, when treated as a standalone regression. In Panel A the outcome measures the proportion of the cohort that speaks English, and cohorts with English, Canadian, and Irish nationalities are excluded from the sample. In Panel B the outcome measures the married proportion of the cohort that is 'in-married' *i.e.* married to someone of the same nationality. Minimum cohort size of 30. Cohort fixed effects are composed of nationality, arrival year (grouped), and tenure (grouped) fixed effects.
All columns present regressions of the outcome variable on a constant, an indicator for whether nationality share is above a threshold, cohort fixed effects, and nationality share itself (allowing for different slopes either side of the threshold). In both panels, Column (1) includes only the subset of cohorts from our main sample that come from countries whose main language is deemed far from English (less than 40% mutual intelligibility), whilst Column (2) uses those who are linguistically near. All covariates except the threshold are demeaned, so the constant (β_0) can be interpreted as an estimate of the mean share speaking English at nationality shares below the threshold. In each case we vary the threshold between .20 and .40, at intervals of .01. We provide the results for the threshold among these which produced a regression with the highest F-statistic when tested against the null of no threshold. The value for this threshold, the estimated F-statistic, and the 5% and 1% critical values for this statistic (Andrews, 1993) are provided at the bottom of the table.

5.4 Discussion of results and limitations

In interpreting our results, we have so far abstracted from the important issue of location choice and the influence that selection might have on our results. We argue that the presence of such selection does not change our ability to draw conclusions about the key trade-offs inherent in our model.

The first question of selection is whether migrants choose where to live based on their individual costs and benefits of interaction, and of the actions they take. The worry might be that any heterogeneity in the cost of switching cultural practices (for example, different costs of learning English or different preferences for maintaining religious practices) will manifest as sorting into different areas. Immigrants with higher costs of switching cultural practices would have a higher relative return from locating in areas where their group continues to maintain their own practices. Such selection would only be a problem for us if we were to imagine that there were discontinuities in the distribution of heterogeneity. Without this, the discontinuity we observe in behavior must be driven by the structure of the game played by individuals in these communities. Sorting by individuals may still mean that migrants with different costs choose to locate in systematically different communities but, as argued by Lazear (1999), this is simply 'a question of timing'. If individuals sort, knowing that after choosing their location their action choice and payoff will depend on those around them, then this is still supportive of the mechanism of our model. The policy implications will differ, however, if some of the difference between the assimilated and segregated communities comes from differences in the costs of assimilation.

The second role for selection relates to who stays in the US. We know from Bandiera et al. (2013) that there was significant churn, with 60-75% of migrants leaving the US. If migrants knew *ex ante* that they were planning to leave the US and go back to their native country, then they would discount the benefits of adopting the practices of the new community.[55] The benefits would now only be felt for a more limited period, whilst the costs of adoption would remain unchanged. This is equivalent to these individuals having the same benefits but higher costs of learning English, and hence the same logic as selection into location applies.

Clearly we do not capture all the richness of the theoretical model. For example, it may be that multiculturalism manifests as the whole community speaking English to one another and sharing some other norms of behavior, but at the same time different immigrant groups maintaining different religious practices, different cuisines, or other different activities. By looking only at the choice to learn English or not we cannot discern such an outcome. Similarly we may miss evidence of 'extreme segregation' because we do not have a rich enough set of outcome variables. An extension of the empirical analysis that looks at choices along more dimensions might allow us to better 'pull apart' the different equilibria. However, the data requirements for such an analysis are strong, including information on social interaction and various action choices among multiple heterogenous populations. Although we can only pick up some of the detail of the theoretical model in our empirical analysis, the variables we do consider show clear evidence that the shape of the relationship is consistent with our predictions.

[55] If the individuals do not know that they might leave, then it cannot influence their decision.

6 Discussion and Concluding Remarks

This paper builds a tractable framework to answer two questions: what social structures can arise in heterogeneous populations, and what environmental features determine which social structure arises. At the center of our analysis is the idea that group behavior and practices, as well as social cohesion, are endogenous. Heterogeneity itself, as well as social relations in heterogeneous populations, greatly depend on the environment.

Understanding how the environment shapes heterogeneous populations is critical for policy design. Integration of immigrant and minority groups receives a lot of political attention, and governments frequently propose and implement policies intended to influence the way minority groups integrate. Our findings paint a nuanced picture of both the relevance and effects of such policies. We briefly consider what our framework implies for two important policies.

First, our model suggests that secularization policies, rather than restraining religion, can actually make groups more likely to maintain diverse religious practices. Secularism prescribes some degree of restriction on religion in public life. In France in 2004 the principle of secularism was applied to ban all conspicuous religious symbols (including veils of any kind) from public schools. Our framework shows that policies which reduce the relevance of cultural practices in everyday life, such as removing religion from state institutions, can enable multiculturalism. That is, it implies that maintaining different religions is easier in a more secular society!

Second, our framework implies that policies which reduce barriers to interaction across groups (e.g. school bussing or desegregation) may result in a response whereby groups not only remain segregated but amplify their differences. Reduced costs of interaction require minority groups to distinguish themselves *even more* if they are to remain segregated. Thus increased opportunities for interaction can lead to 'secondary' differences where minority groups begin to create new distinguishing features and emphasise previously insignificant behaviors as culturally important.

Our framework does not rule out a role for policy however. The key welfare tension in the model comes from the inability of group members to coordinate, which can lead to 'inefficient segregation'.[56] A failure to coordinate among immigrants allows segregation to arise in situations where a collective move to assimilation or multiculturalism would be a Pareto improvement. This is because when groups are segregated under Nash equilibrium, a minority

[56]See Online Appendix B for an expanded welfare analysis.

individual does worse by switching culture and joining the majority group, even though the minority group as a whole might do better by adopting majority culture. Policy can influence this by reducing the costs of switching culture for at least some minority individuals, thus breaking down the segregation equilibrium. Schooling policy provides an example of this, where the teaching of national values to the children of immigrants makes it easier for them to integrate with natives, thus making it harder for their parents to remain segregated (see Alesina and Reich (2015), Bandiera et al. (2015) and references therein).

Of course, the costs of switching culture and the ubiquity of cultural and religious practices in daily life could be influenced not only by governments by also by individuals and groups. 'Group leaders' might be able to influence these parameters to achieve their individual aims. For example religious leaders might care most about preserving religious adherence, and attempt to adjust the cost of switching practices or increase the ubiquity of religious activities, even at the cost of lowering the utility of group members. Alternatively, the costs of switching culture and the relevance of cultural and religious practices could emerge endogenously in a decentralized model. This paper takes a key step in understanding endogeneity of heterogeneity.[57] However, there remain many important avenues for further work in this direction.

A second important direction for future research would be to allow for not just more than two types of individual but different dimensions of types. For example, we may have two different nationality groups with *ex ante* different practices, but each nationality group may itself be composed of a mix of people of two different religions which have their own associated practices. An illustration is provided by the Wars of the Three Kingdoms, which saw Protestants in England, Scotland, and Ireland (before the formation of the United Kingdom) fighting together against Catholics from their own countries. Adapting our framework with such an addition may provide answers to questions about when and why one divide (for example religion) becomes more important than another (for example ethnicity) in different societies.

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Appendix A

Proof of Proposition 1

In a Nash equilibrium all players of the same type must play the same action set. Suppose not, and that two players of the same type, i and j, play different action sets. One player, without loss of generality player i, must do weakly worse than the other. Then i can do strictly better by mimicking j's strategy and also forming a tie with j. Forming a tie with j ensures that player i either has an additional tie compared to j's previous strategy, or that the value i receives from the tie between himself and j is higher than the value j would have been receiving from this tie, since they now coordinate on more actions. Hence in equilibrium i and j must play the same actions. In equilibrium each player must also link with all others who adopt the same action set since the value of such a tie is strictly positive. Thus all individuals in the same group form a tie in equilibrium.

Suppose $1-\alpha < L$. Suppose there is (at least) one tie between different types in equilibrium. If the two types do not have culture in common then the payoff from this tie is at most $1-\alpha-L < 0$, and the individual can do strictly better by dropping the tie. In equilibrium the two groups must have culture in common for there to be a tie between groups. Suppose the two groups have culture but do not have non-cultural practices in common. Then a player from the weakly smaller group can do strictly better by adopting the non-cultural action of the weakly larger group and linking accordingly. Therefore, if there is a tie between groups in equilibrium, all individuals must play the same action set and thus all individuals must be linked.

Suppose there are no ties between different types in equilibrium. If the two types adopt the same cultural practices but do not have non-cultural practices in common, then a player from the weakly smaller group can do strictly better by adopting the non-cultural action of the weakly larger group and instead linking with that group. If the two types adopt the same action set, then they would have strictly higher utility by linking across groups. Thus if there are no ties between different types, in equilibrium the two types must adopt different cultural actions. Further, each type must adopt its own cultural action since, if not, a type M must be doing weakly better by playing x_m and linking with his group than playing x_M and linking with the other group. But then a type m must do strictly better by playing x_m, linking with the other group and not paying the cost of switching culture. A contradiction.

We have ruled out all states apart from assimilation and segregation states as possible equilibria. Assimilation is an equilibrium under all parameter values since we assumed $(1-L)(n-1)-c > \max\{0, (1-\alpha-L)(n-1)\}$, and so all deviations by an individual do worse. Segregation is an equilibrium if $(1-L)(n_m-1) \geq (1-L)(n-n_m)-c$. It is straightforward to see that under this condition all deviations by an individual do worse.

Suppose $1-\alpha \geq L$. Suppose there are no ties between different types in equilibrium. If the two types adopt the same cultural practices but do not have non-cultural practices in common, then a player from the weakly smaller group can do strictly better by adopting the non-cultural action of the weakly larger group and instead linking with that group. If the two types adopt the same action set, then they would have strictly higher utility by linking across groups. The remaining possibility is that the two types adopt different cultural actions. If the two types adopt the same non-cultural action the value of a tie with the other type is $1-\alpha-L \geq 0$ and so this is not an equilibrium. Thus different types must adopt different cultural and non-cultural actions. Each type must also adopt their own cultural action, by the above.

Suppose there is (at least) one tie between different types in equilibrium. If the two types have cultural practices in common then, by the proof above, they must also have non-cultural actions in common and thus all individuals must be linked. If the two types do not have cultural practices in common, then suppose they also do not have non-cultural practices in common. Then the value of a tie between types is strictly negative and the individual with a tie to a different type does strictly better by dropping it. If the two types do not have cultural practices in common, then the remaining possibility is that they do have non-cultural practices in common. Then the value of a tie with the other type is weakly positive and all individuals must be linked.

We have ruled out all states apart from assimilation, segregation, and multiculturalism as possible equilibria. Assimilation is an equilibrium under all parameter values. Given the assumption $(1-L)(n-1)-c > (1-\alpha-L)(n-1)$, any deviation from assimilation by a lone individual results in a strictly lower payoff.

Segregation is an equilibrium when no player wants to deviate. No player wishes to deviate, play the action set of the other group, and link only with the other group if and only if

$$(1-L)(n_m-1) \geq (1-L)(n-n_m)-c$$

or deviate, switch non-cultural action, and link with the other group as well as their own type if and only if

$$(1-L)(n_m-1) \geq (\alpha-L)(n_m-1)+(1-\alpha-L)(n-n_m).$$

It is straightforward to see that under these conditions all other deviations by an individual away from segregation do worse. Multiculturalism is an equilibrium when no player wishes to mimic the strategy of the other group if and only if

$$(1-L)(n_m-1)+(1-\alpha-L)(n-n_m) \geq (1-L)(n-n_m)+(1-\alpha-L)(n_m-1)-c.$$

It is straightforward to see that under these conditions all deviations by an individual do worse.

Proof of Proposition 2

The process described defines an aperiodic and irreducible Markov Chain; it therefore has a unique stationary distribution. We briefly describe our notation and method of computing the stationary distribution as $\epsilon \to 0$. We refer to the strict Nash equilibria of the game, denoted here by σ, as *nodes.* There is a minimum number of players required to make a mistake in their strategy for there to be a positive probability of transiting from one strict Nash equilibrium, call it σ', to another strict Nash equilibrium, say σ, by the best response dynamics. We refer to this minimum mistake transition from node σ' to node σ as the *link* from σ' to σ and the minimum number of players required to make a mistake as its *weight.* A path from node σ' to σ is a sequence of directed links starting at σ' and connecting to σ. A $\sigma - tree$ is graph with no cycles such that for each strict Nash equilibrium $\sigma' \neq \sigma$ a unique path connects it (directly or indirectly) to σ. The *stochastic potential* of σ is the total weight of the $\sigma - tree$ whose links sum to the lowest total weight. A *stochastically stable state* is a state with positive probability in the stationary distribution when $\epsilon \to 0$. The stochastically stable states are the strict Nash equilibria with minimum stochastic potential. This result and details can be found in Young

(1993).[58]

To find the $\sigma - tree$ for each strict Nash equilibrium σ would involve enumerating a lot of possibilities. We use a tree-pruning argument to simplify things. The proof here is given for the parameter range $1-\alpha < L$. The result for the reverse inequality is similar and given in Online Appendix B.

Lemma 1 *Partition the strict Nash equilibria into three sets: segregation; assimilation to M ('M-assimilation'); and assimilation to m ('m-assimilation'). Such a set is denoted Σ_x, and $x \in \{s, M, m\}$ respectively indexes the set. All equilibria in a set, $\sigma \in \Sigma_x$, have the same stochastic potential.*

Proof. Equilibria in a given set are symmetric so, for any $\sigma - tree$, an equivalent $\sigma - tree$ can be constructed with the same total weight for any other equilibrium in the same set.

Lemma 2 *A strict Nash equilibrium $\hat{\sigma} \in \Sigma_x$ has lower stochastic potential than a strict Nash equilibrium $\tilde{\sigma} \in \Sigma_y$, $y \neq x$, if the link from $\tilde{\sigma}$ to $\hat{\sigma}$ has lower weight than the link from $\hat{\sigma}$ to σ for all $\sigma \in \Sigma_z$, for all $z \neq x$.*

Proof. Take a tree which defines the stochastic potential of node $\tilde{\sigma}$. Denote by $p_{\sigma_1\tilde{\sigma}}$ the path from some node denoted σ_1 to $\tilde{\sigma}$ in such a tree. Next find the link from $\tilde{\sigma}$ to $\hat{\sigma}$ and denote this link by $l_{\tilde{\sigma}\hat{\sigma}}$. Adding this link to the tree forms a graph that contains paths from all nodes to $\hat{\sigma}$, since the graph contains the path $p_{\sigma_1\tilde{\sigma}} + l_{\tilde{\sigma}\hat{\sigma}}$ for all σ_1. Observe that the tree to $\tilde{\sigma}$ must involve a link from the chosen $\hat{\sigma}$ equilibrium to some other node σ_2, denoted $l_{\hat{\sigma}\sigma_2}$. This link, $l_{\hat{\sigma}\sigma_2}$ is incorporated into a path from σ_1 as follows: $p_{\sigma_1\hat{\sigma}} + l_{\hat{\sigma}\sigma_2} + p_{\sigma_2\tilde{\sigma}} + l_{\tilde{\sigma}\hat{\sigma}}$ (where it can be that $\sigma_1 = \hat{\sigma}$, so the path starts at $\hat{\sigma}$, and/or $\sigma_2 = \tilde{\sigma}$ so the link from $\hat{\sigma}$ in the original tree is directly to $\tilde{\sigma}$). It is clear that by deleting the link $l_{\hat{\sigma}\sigma_2}$ there is still a path from any node to the node $\hat{\sigma}$. If the weight of the link added ($l_{\tilde{\sigma}\hat{\sigma}}$) is lower than the weight of the link deleted (some $l_{\hat{\sigma}\sigma_2}$) then the lowest weight tree to $\hat{\sigma}$ has lower total weight than the lowest weight tree to $\tilde{\sigma}$.

To show that one need only examine links from $\hat{\sigma}$ out of the set Σ_x, observe the following. If the link $l_{\hat{\sigma}\sigma_2}$ is to some $\sigma_2 \in \Sigma_x$, then the path $p_{\sigma_2\tilde{\sigma}}$ must include a link from a node in Σ_x, denoted σ_3 to a node, denoted σ_4, where $\sigma_4 \in \Sigma_y$, $y \neq x$, (where it could be that $\sigma_3 = \sigma_2$). The path $l_{\hat{\sigma}\sigma_2} + p_{\sigma_2\tilde{\sigma}}$ must include a path from $\hat{\sigma}$ to σ_3 that links only nodes in Σ_x. Now delete the link from σ_3 to $\sigma_4 \in \Sigma_y$, $l_{\sigma_3\sigma_4}$, and reverse all links on the path from $\hat{\sigma}$ to σ_3. This changes the

[58]This result also requires that the process without experimenting converges almost surely to a strict Nash equilibria. This is straightforward and, for completeness, is found in Online Appendix B.

path $p_{\sigma_1\hat{\sigma}} + l_{\hat{\sigma}\sigma_2} + p_{\sigma_2\tilde{\sigma}} + l_{\tilde{\sigma}\hat{\sigma}}$ into the following paths $p_{\sigma_1\hat{\sigma}}$, $p_{\sigma_3\hat{\sigma}}$, and $p_{\sigma_4\tilde{\sigma}} + l_{\tilde{\sigma}\hat{\sigma}}$. By deleting the link $l_{\sigma_3\sigma_4}$ there is still a path from any node on the path $p_{\sigma_1\hat{\sigma}} + l_{\hat{\sigma}\sigma_2} + p_{\sigma_2\tilde{\sigma}} + l_{\tilde{\sigma}\hat{\sigma}}$ to the node $\hat{\sigma}$. Similarly if the path from any node j includes any node on the path $p_{\sigma_1\hat{\sigma}} + l_{\hat{\sigma}\sigma_2} + p_{\sigma_2\tilde{\sigma}} + l_{\tilde{\sigma}\hat{\sigma}}$ then there continues to be a link to $\hat{\sigma}$. Observe that reversing a link between two nodes in a given set does not change the weight of the link. Thus if the weight of link $l_{\sigma_3\sigma_4}$ is higher than that of link $l_{\tilde{\sigma}\hat{\sigma}}$ then the lowest weight tree to $\hat{\sigma}$ has lower total weight than the lowest weight tree to $\tilde{\sigma}$. To conclude the proof, note that equilibria are symmetric so the path $l_{\sigma_3\sigma_4}$ will have an analogous path from $\hat{\sigma}$. $\square$

To simplify notation, we denote an equilibrium where both types play (x^M, y^A) by (M, A) and analogously for other assimilation equilibria. We denote an equilibrium where type M play (x^M, y^A) and type m play (x^m, y^B) by $(M, A; m, B)$, and analogously for other segregation equilibria. The lowest weight link between each group, denoted ϕ, is detailed in Table A1. For now we allow the weight of a link to be a real number, $\phi \in \mathbb{R}$, and address the integer nature of mistakes at the end of the proof. Given Lemmas 1 and 2, the following statements follow in a straightforward manner from Table A1.

M-assimilation has lower stochastic potential than segregation when $\phi_{sM} < \phi_{Ms}$.

Take the lowest weight tree to any segregation strict Nash equilibrium. The lowest weight link from this node to some M-assimilation equilibrium is ϕ_{sM}. It can be seen from Table A1 that the weight of any link from M-assimilation to m-assimilation, or to segregation, is at least ϕ_{Ms}. Thus the result follows from Lemmas 1 and 2.

Segregation has lower stochastic potential than M-assimilation when $\phi_{Ms} < \phi_{sM}$.

Take the lowest weight tree to any M-assimilation equilibrium. The lowest weight link from this node to some segregation equilibrium is ϕ_{Ms}. Any link from segregation to M-assimilation, or to m-assimilation, has weight at least ϕ_{sM}. By Lemma 1 and 2 the statement holds.

Segregation has lower stochastic potential than m-assimilation when $\phi_{Ms} < \phi_{sM}$.

The lowest weight link from any m-assimilation equilibrium to segregation is ϕ_{Ms}. The result follows as above.

M-assimilation has equal stochastic potential to m-assimilation when $\phi_{sM} < \phi_{sm} < \phi_{Ms}$. Take the lowest weight tree to any M-assimilation equilibrium. The lowest weight link from this node to some m-assimilation equilibrium is ϕ_{Ms}, since $\phi_{sm} < \phi_{Ms}$. Any link from m-assimilation to M-assimilation, or to segregation, has weight at least ϕ_{Ms}, since $\phi_{sM} < \phi_{Ms}$. By Lemma 1 and Lemma 2, m-assimilation has weakly lower stochastic potential than M-assimilation. Starting from the lowest weight tree to any m-assimilation equilibrium repeat the process to show M-assimilation has weakly lower stochastic potential than m-assimilation.

M-assimilation has lower stochastic potential than m-assimilation when $\phi_{sM} < \phi_{Ms} < \phi_{sm}$. Take the lowest weight tree to any m-assimilation equilibrium. Suppose this lowest weight tree includes a link from one of the segregation equilibria to (m, A) or (m, B). The minimum weight link from some segregation equilibrium to some m-assimilation equilibrium is ϕ_{sm}. Delete this link and form a link from the same segregation equilibrium, by the lowest weight link, to either (M, A) or (M, B), which will have weight ϕ_{sM}. The new graph has strictly lower total weight. Now form a link from the m-assimilation equilibrium to the same node, either (M, A) or (M, B), which will be weight ϕ_{Ms}, and delete a link from the node either (M, A) or (M, B), which are all of weight at least ϕ_{Ms}. Thus we have a graph from all nodes to (M, A) or (M, B) with strictly lower total weight than the tree to m-assimilation.

Suppose instead the lowest weight tree to any m-assimilation equilibrium has no link from a segregation equilibria to (m, A) or (m, B). Then there must be a link from either (M, A) or (M, B) to either (m, A) or (m, B) in order to have a tree to m-assimilation. Now a link from either (M, A) or (M, B) has the same weight to either (m, A) or (m, B) so we can replace it with a link to the chosen m-assimilation equilibrium while maintaining the tree and without affecting the weight of the tree. Now we delete this link and form the reverse link from the chosen m-assimilation equilibrium to the node (M, A) or (M, B) whose link to m-assimilation was just deleted. The graph now has strictly lower weight and maintains a link to all nodes.

Above we suppose $\phi \in \mathbb{R}$. Since our population is discrete, the minimum weights are in fact the lowest integer greater than or equal to a given ϕ. Given the discrete nature of the population the equilibrium is guaranteed to be unique when $\phi_{Ms} < \phi_{sM}$ if $\phi_{sM} - \phi_{Ms} \geq 1$

and when $\phi_{Ms} > \phi_{sM}$ if $\phi_{Ms} - \phi_{sM} \geq 1$. Since $\phi_{sM} \gtreqless \phi_{Ms}$ implies $n_m \gtreqless n - c/(1-L)$, $\phi_{sM} - \phi_{Ms} \geq 1$ is equivalent to $n_m \geq n - c/(1-L) + 1 = \delta^* + 1$, and $\phi_{Ms} - \phi_{sM} \geq 1$ is equivalent to $n_m \leq n - c/(1-L) - 1 = \delta^* - 1$. M-assimilation has lower stochastic potential than m-assimilation when $\phi_{sm} > \phi_{Ms}$, allowing for discreteness requires $\phi_{sm} - \phi_{Ms} \geq 1$ which implies $n_m \leq c/(1-L) - 1$. This final inequality is Assumption 2.

Table A1: The lowest weight link from a node in one set to a node in a different set, for $1-\alpha < L$ and $\alpha \gtreqless L$.

Sets linked	Minimum number of mistakes to transit between sets is min. ϕ satisfying:	Notes	Weight denoted
segregation to M-assimilation	$(1-L)(n_M+\phi)-c \geq (1-L)(n_m-\phi-1)$	e.g. from $(M,A;m,B)$ to (M,A). This involves type m making a mistake of (M,A).	ϕ_{sM}
segregation to m-assimilation	$(1-L)(n_m+\phi)-c \geq (1-L)(n_M-\phi-1)$	e.g. from $(M,A;m,B)$ to (m,B). This involves mistakes of (m,B).	ϕ_{sm}
M-assimilation to segregation	$(1-L)\phi \geq (1-L)(n-\phi-1)-c$	e.g. from (M,A) to $(M,A;m,B)$. This involves mistakes of (m,B).	ϕ_{Ms}
m-assimilation to segregation	$(1-L)\phi \geq (1-L)(n-\phi-1)-c$	e.g. from (m,B) to $(M,A;m,B)$. This involves mistakes of (M,A).	ϕ_{Ms}
m-assimilation to M-assimilation	$(1-L)\phi \geq (1-L)(n-k-1)-c$ and $(1-L)(n_M+\phi')-c \geq (1-L)(n_m-\phi'-1)$	e.g. from (m,A) to (M,A). Type M want to transition after at least ϕ mistakes of (M,A). Once all type M have moved, type m want to transition if at least ϕ' mistakes are made (by type m). The minimum number of mistakes must satisfy both inequalities.	$\max\{\phi_{Ms},\phi_{sM}\}$
M-assimilation to m-assimilation	$(1-L)\phi \geq (1-L)(n-k-1)-c$ and $(1-L)(n_m+\phi')-c \geq (1-L)(n_M-\phi'-1)$	e.g. from (M,A) to (m,A). Type m want to transition after at least ϕ mistakes of (m,A). Once all type m have moved, type M want to transition if at least ϕ' mistakes are made (by type M).	$\max\{\phi_{Ms},\phi_{sm}\}$

Online Appendix B Additional discussion of the model and proofs

Extending the model to action choices along more than two dimensions

We have modeled action choice along two dimensions. In reality, individuals make many more than two decisions. The minority group may initially speak one language and the majority group another, the minority group may participate in certain religious practices and the majority group others, the culture of the minority group may permit alcohol while that of the majority group forbids it, and so on. The baseline model condenses these many choices into just two: a choice of cultural action, which represents choice over practices for which there is type-specific cost, and a choice of non-cultural action, which represents choice over practices which have no type specific cost. Very little intuition is lost through this abstraction. Here we relax this assumption and highlight the novel features which arise.

Let x_{i1} denote i's choice of language, which is chosen from the set $\{x_1^m, x_1^M\}$, where x_1^m is the language of the minority group and x_1^M is the language of the majority. If two individuals speak the same language then the benefit from interaction increases by α_1. Suppose the cost to switching language is c_1. Let x_{i2} denote i's choice to drink alcohol or not, chosen from the set $\{x_2^m, x_2^M\}$, where group m permits alcohol and group M does not. Suppose α_2 is the importance of having this in common in terms of enabling interaction and economic exchange. Let c_2 be the cost of switching. Generally let $(x_{i1}, \ldots, x_{iQ})$ denote the Q 'cultural choices' of individual i. The set $(x_1^m, \ldots, x_Q^m)$ contains the *ex ante* cultural practices of group m, and the set $(x_1^M, \ldots, x_Q^M)$ the *ex ante* cultural practices of group M. Some cultural practices may be very costly to switch away from and others less so, represented by the cost of switching for each practice, c_r, $r \in \{1, \ldots, Q\}$. Some practices may be very relevant to interaction and others less so, represented by α_r, $r \in \{1, \ldots, Q\}$. Non-cultural actions are modeled similarly. Individual i chooses R non-cultural actions, $(y_{iQ+1}, \ldots, y_{iQ+R})$, where for each action there is a choice, $\{y_{Q+1}^A, y_{Q+1}^B\}, \ldots, \{y_{Q+R}^A, y_{Q+R}^B\}$, where the choice is denoted by the superscripts A and B. Any non-cultural choice has an associated cost of zero, $c_r = 0$. The importance of each practice in terms of social interaction is given by α_r, $r \in \{Q+1, \ldots, Q+R\}$. We normalize these to sum to one.

$$\sum_{r=1}^{Q+R} \alpha_r = 1.$$

Each individual chooses social ties as before, but now chooses a larger set of actions

$(x_{i1}, \ldots, x_{iQ}, y_{iQ+1}, \ldots, y_{iQ+R})$. The utility function is now

$$u_k(s_i, s_{-i}) = \sum_{j=1}^{n} \left[\sum_{r \in \{1,\ldots,Q\}} \alpha_r \pi_r(x_{ir}, x_{jr}) + \sum_{r \in \{Q+1,\ldots,Q+R\}} \alpha_r \pi_r(y_{ir}, y_{jr}) - L \right] g_{ij} - \sum_r c_k(x_i r) \quad (4)$$

where

$$\pi(x_{ir}, x_{jr}) = \begin{cases} 1 & \text{if } x_{ir} = x_{jr} \\ 0 & \text{if } x_{ir} \neq x_{jr} \end{cases}$$

$$\pi(y_{ir}, y_{jr}) = \begin{cases} 1 & \text{if } y_{ir} = y_{jr} \\ 0 & \text{if } y_{ir} \neq y_{jr} \end{cases}$$

and for individual i of type $k \in \{M, m\}$

$$c_k(x_i r) = \begin{cases} 0 & \text{if } x_{ir} = x_r^k \\ c_r & \text{if } x_{ir} \neq x_r^k \end{cases}$$

The Nash equilibria and comparative statics with multiple dimensions are analogous to those in the main model presented. We highlight only the notable richness that adding more dimensions brings. Under similar conditions to the main model, assimilation outcomes are Nash equilibria, where all individuals form a tie, all adopt the same practices where one group adopts all the cultural practices of the other group. There now exist what we refer to as 'melting pot' equilibria, a type of assimilation equilibrium where all individuals form a tie and all adopt the same practices (both cultural and non-cultural), but the action choices could be a mix of the *ex ante* cultural practices of the two groups. Thus the *ex post* culture that emerges is a mix of the *ex ante* different cultures (Kuran and Sandholm, 2008). There are also multiculturalism equilibria, similar to main model, whereby all individuals form a tie, have all non-cultural practices in common, but the different groups maintain at least some of their different cultural practices. There is some added richness here which does not appear when there are only two action choices: the smaller the minority group the fewer minority traits they can sustain in a multiculturalism equilibrium. There is a sequence of multiculturalism equilibria, from the case where each group maintains all their own cultural actions, to fewer and fewer of one group's traits being retained, moving towards assimilation. The only other Nash equilibria are segregation equilibria: individuals form ties only within their group and not to the other group, and adopt sufficiently different action sets (this includes versions of the 'extreme segregation' outcome).[59] There is also added richness here: the smaller the minority group, the more they must differentiate their practices in order for segregation to be a Nash equilibrium. This is consistent with empirical findings (Bisin et al., 2013) and theoretical findings (Bisin and Verdier, 2000) that smaller minority groups must exert more effort to retain their diverse traits.

Discussion of alternative models of link formation

As mentioned, the model of link formation presented here is not the only way to model social interactions. For example, the Nash equilibria in this paper all satisfy the definition of *pairwise stability*, an important measure of stability in network formation (Jackson and Wolinsky, 1996). In fact, a two-sided link formation model, related to pairwise stability, where an individual could delete any number of links and form any number of mutually agreed upon links would produce the same outcomes as the link formation model we use. There are many ways to model how individuals form ties and in any framework a choice must be made. The key consideration in choosing the model of link formation here was to avoid a multitude of equilibria (often a feature of social interaction models) that add complication without giving additional intuition with respect to the questions we seek to answer.

Another way of modeling social interaction would be to add a cap on the number of ties an individual can form, or to assume decreasing returns in the number of ties. With such assumptions, the qualitative results would remain in smaller communities and we would see more segregation in larger populations where the cap on the number of contacts becomes relevant to decision making.

Welfare results and discussion

We will consider which states are Pareto efficient and which would be chosen by a social planner, where the social planner maximizes the sum of utilities in the population

$$\sum_i u_k(s_i, s_{-i}).$$

We highlight three findings.

[59] The segregation equilibria have the feature that, although a group will mainly adopt its own cultural practices, it may be that a group adopts a few cultural practices of the other group. This is a feature of examining Nash equilibria in a coordination game.

First, multiculturalism is Pareto efficient under the same parameters that multiculturalism will persist in the long run. Similarly segregation is Pareto efficient exactly when it will persist in the long run.[60] That is, segregation and multiculturalism are likely to emerge in the long-run exactly when this outcome is in the minority group's best interest.[61]

Second, assimilation is always Pareto efficient. A group can do no better than when the other group assimilates, since this maximizes interaction and the other group pays the cost of switching culture. This suggests that when a group is able to do so, it will put pressure on other groups to do the assimilating. Calls for immigrants to adopt local languages and cultural practices by the public or by politicians are thus unsurprising. In contrast, a social planner will not always choose assimilation.

The third point to highlight is the result of Proposition 3 (below). This reveals a tension between when segregation is a Nash equilibrium, when it is optimal for the minority group, and when it is optimal for the social planner. Proposition 3 says that segregation is a Nash equilibrium under a larger set of parameters than those for which it is Pareto optimal, and segregation is Pareto optimal under a larger set of parameters than those for which the social planner would choose to implement segregation. The reason for this tension is that segregation is a Nash equilibrium when, given the population is segregated, no individual from the minority group does better by switching culture and joining the majority group. However, even if a minority individual in this situation prefers to stick with minority culture, it could be that the minority group as a whole would do better by adopting majority culture. The social planner chooses segregation under a yet smaller set of parameters, since the social planner also incorporates the costs of segregation to the majority group. This same reasoning applies to multiculturalism under the parameters $1-\alpha \geq L$.

Let δ^W denote the threshold size of the minority group above which segregation is chosen by the social planner, let δ^P denote the threshold above which segregation is Pareto efficient, and δ is the threshold above which segregation is a Nash equilibrium, given in Proposition 1. Analogously for η^W, η^P, and η for multiculturalism. Observe in Proposition 3 that a social planner would choose multiculturalism when $1-\alpha \geq L$ and the minority group is large, again lending support to the proposed ideal of a multicultural society.

[60] The only difference is that we do not need to account for discreteness as in Proposition 2.

[61] Excluding the case where the majority group assimilate to minority culture, which the minority group would prefer but cannot influence.

Proposition 3 *When* $1 - \alpha < L$*:*

$$\delta^W > \delta^P > \delta.$$

When $1 - \alpha \geq L$*:*

$$\eta^W > \eta^P > \eta.$$

Proof of Proposition 3.

Any assimilation state is always Pareto efficient since any individual whose cultural action is played attains his maximum payoff, doing strictly worse under any other state.

Segregation is Pareto efficient when (5) holds and $1-\alpha < L$. Each individual in the minority group gets a strictly higher payoff from segregation than assimilation to majority culture when

$$(1 - L)(n_m - 1) > (1 - L)(n - 1) - c$$

rewritten

$$n_m/n > 1 - c/n(1 - L). \tag{5}$$

It is immediate to see that individuals in the minority group get a higher payoff from segregation than any other state where only the majority cultural action is played. Symmetrically for the majority group. It remains to show that some individual does strictly worse in any state, other than segregation, where at least two individuals adopt different cultural actions. Since $1 - \alpha < L$, any individual who links with someone adopting a different cultural action does strictly worse. If fewer individuals play the minority cultural action, then those remaining playing the minority action must be strictly worse off (symmetrically for the majority cultural action). The remaining possibility is that the size of each group playing either cultural action remains the same, but some majority and minority individuals switch strategy. Clearly some individuals do strictly worse.

Multiculturalism is Pareto efficient when $1 - \alpha \geq L$ and the minority group does strictly worse by assimilating

$$(1 - L)(n_m - 1) + (1 - \alpha - L)n_M > (1 - L)(n - 1) - c$$

rewritten

$$n_m/n > 1 - \frac{c}{\alpha n}.$$

The proof follows from above, also noting that under $1 - \alpha \geq L$ individuals do weakly better by all adopting the same non-cultural action and linking.

Assimilation by the minority gives higher welfare than any other state in which a single cultural action is played. When $1 - \alpha < L$, in any state that maximizes welfare with two cultural actions being played, any two individuals playing the same cultural action must also play the same non-cultural action and must link, and any two individuals playing different cultural actions must not link. Segregation gives weakly higher total welfare than assimilation by the minority when

$$n_m(1 - L)(n_m - 1) + n_M(1 - L)(n_M - 1) \geq n_m[(1 - L)(n - 1) - c] + n_M(1 - L)(n - 1)$$

rewritten

$$n_m/n \geq 1 - c/2n(1 - L).$$

It remains to show that total welfare is weakly lower when some other combination of individuals play different cultural actions. Suppose a smaller group of minority individuals, $n_m - n_1$, play the minority cultural action and the rest, n_1, play the majority cultural action, for $n_1 \in \{1, \ldots, n_m - 1\}$. Total welfare is higher than under segregation if

$$\begin{aligned}(n_m - n_1)(1 - L)(n_m - n_1 - 1) + (n_M + n_1)(1 - L)(n_M + n_1 - 1) - n_1 c \\ \geq n_m(1 - L)(n_m - 1) + n_M(1 - L)(n_M - 1)\end{aligned}$$

rewritten

$$(n_M + n_1)/n - c/2n(1 - L) \geq n_m/n.$$

A contradiction. The same applies if a smaller group of majority individuals play the majority cultural action. Finally it is clear that each type playing their respective cultural action maximizes welfare in this situation.

Multiculturalism maximizes the social planner's problem when $1 - \alpha \geq L$ and it gives higher total welfare than assimilation by the minority, that is

$$n_m(1-L)(n_m-1)+n_M(1-L)(n_M-1)+n_m(1-\alpha-L)n_M \\ +n_M(1-\alpha-L)n_m \geq n_m[(1-L)(n-1)-c]+n_M(1-L)(n-1)$$

rewritten

$$n_m/n \geq 1-c/2\alpha n.$$

The result follows similarly to above. □

Heterogeneous costs of switching culture.

Suppose for members of the immigrant group that the costs of switching culture are given by

$$c_{1m} \leq c_{2m} \leq \ldots \leq c_{n_m m}$$

and similarly for the native group

$$c_{1M} \leq c_{2M} \leq \ldots \leq c_{n_M M}.$$

Let us consider the case where $1-\alpha < L$. Then individuals will not form a tie if they have only non-cultural actions in common.

Assimilation (to immigrant or native culture) is always an equilibrium provided that the following condition, analogous to the previous *no man is an island* assumption, holds:

$$(1-L)(n-1)-c_h > 0 \quad \forall h \in \{1m, 2m, \ldots, n_m m, 1M, 2M, \ldots, n_M M\}.$$

Suppose all individuals in the population adopt the same cultural action, then by the same argument as the proof of Proposition 1, in equilibrium all must adopt the same non-cultural action and form a link. Thus assimilation states are Nash equilibria under all parameters and they are the only Nash equilibria where all individuals adopt the same cultural action.

Suppose some individuals in the population adopt different cultural actions. Let n_1 be the size of the group adopting x_m and n_2 be the size of the group adopting x_M. In equilibrium those playing the same cultural action must play the same non-cultural action and link otherwise there are profitable deviations.

Suppose $n_1 \geq n_2$. Then any minority individuals playing x^M do strictly better by switching to play x_m and so in any Nash equilibrium all minority individuals must be in group n_1. No minority individual wishes to deviate. A majority individual in group n_1 with cost c_h does not want to deviate if

$$(1-L)(n_1-1) - c_h \geq (1-L)n_2$$

and a majority individual in group n_2 with cost c_h' does not want to deviate if

$$(1-L)(n_2-1) \geq (1-L)n_1 - c_h'.$$

Note that the final two conditions hold only if all majority individuals in group n_1 have a lower cost of switching, c_h, than all individuals in group n_2.

Instead suppose $n_2 \geq n_1$. In any Nash equilibrium all majority individuals must be in group n_2. No minority individual with cost c_h wishes to deviate from group n_2 if

$$(1-L)(n_2-1) - c_h \geq (1-L)n_1$$

No minority individual with cost c_h' wishes to deviate from group n_2 if

$$(1-L)(n_1-1) \geq (1-L)n_2 - c_h'.$$

This is the same as above, although it is not guaranteed that it will hold even if the whole minority group is part of n_1. As with the homogeneous cost case segregation only holds when the minority group is large enough.

Additional proofs for Proposition 2

Proof that the Markov process converges almost surely to a strict Nash equilibrium

We show there exists no recurrence class other than the strict Nash equilibria. To do so it suffices to show that from any state there are a finite number of positive probability events which lead to a strict Nash equilibrium.

Suppose $n_1 \leq n$ players play (M, A), $n_2 \leq n$ players play (m, A), $n_3 \leq n$ players play (M, B) and $n_4 \leq n$ players play (m, B), where either type can be playing either action. With positive probability a given type M is selected to update his strategy. Suppose the type M does (at least weakly) best by playing (M, A), and chooses (M, A). Suppose next a different individual of type M is selected to update his strategy; again this occurs with positive probability. This individual must strictly prefer (M, A) since the payoff from playing (M, A) relative to other actions has strictly increased and it was weakly optimal for a type M in the previous period. Let this continue such that all type M are selected and no type m. All type M now play (M, A). Next a given type m is selected to update his strategy with positive probability. His optimal strategy may be either (M, A), (m, A), or (m, B). He chooses one of these actions. Let this continue until all type m have been selected to update their strategy. The action chosen by the first type m agent selected to update his strategy is strictly best for all type m that follow. The process arrives at a strict Nash equilibrium since all type M play (M, A) and all type m play one of either (M, A), (m, A), or (m, B). Thus we have either assimilation by type m, segregation, or multiculturalism. The same argument holds if instead the initial type M selected to update his strategy selects (M, B). If instead the initial type M selected to update his strategy selects (m, A), then all type M or m that follow must do strictly best by playing (m, A). The result follows. Similarly if instead the initial type M selected does best by playing (m, B).

Proof of Proposition 2 for parameters $1 - \alpha > L$.

Lemma 1 becomes Lemma 3:

Lemma 3 *Partition the strict Nash equilibria into four sets: segregation; multiculturalism; M-assimilation; and m-assimilation. Such a set is denoted Σ_x and $x \in \{s, u, M, m\}$ respectively*

Table B1: Finding the lowest weight link from a node in one set to a node in a different set, for $1-\alpha > L$ and $\alpha \gtreqless L$.

Sets linked	Minimum number of mistakes to transit between sets is min. ϕ satisfying:	Notes	Weight denoted
M-assimilation to multiculturalism	$(1-L)\phi + (1-\alpha-L)(n-\phi-1) \geq (1-L)(n-\phi-1) + (1-\alpha-L)\phi - c$, which implies $\phi \geq \frac{n-1}{2} - \frac{c}{2\alpha}$	e.g. from (M, A) to (MA, mA).	ϕ_{Mu}
m-assimilation to multiculturalism	As above, $\phi \geq \frac{n-1}{2} - \frac{c}{2\alpha}$	e.g. from (m, A) to (MA, mA).	ϕ_{Mu}
M-assimilation to segregation	$(1-L)\phi \geq (1-L)(n-\phi-1) - c$, which implies $\phi \geq \frac{n-1}{2} - \frac{c}{2(1-L)}$	e.g. from (M, A) to (MA, mB).	ϕ_{Ms}
m-assimilation to segregation	As above, $\phi \geq \frac{n-1}{2} - \frac{c}{2(1-L)}$	e.g. from (m, A) to (MB, mA).	ϕ_{Ms}
multiculturalism to M-assimilation	$(1-L)(n_M+\phi) + (1-\alpha-L)(n_m-\phi-1) - c \geq (1-L)(n_m-\phi-1) + (1-\alpha-L)(n_M+\phi)$, which implies $\phi \geq \frac{n_m-n_M-1}{2} + \frac{c}{2\alpha}$	e.g. from (MA, mA) to (M, A). [The alternative link, say (MA, mA) to (M, B), has weight at least $\frac{n_M-1}{2} + \frac{1-\alpha-L}{2(1-L)}n_m$.]	ϕ_{uM}
multiculturalism to m-assimilation	$(1-L)(n_m+\phi) + (1-\alpha-L)(n_M-\phi-1) - c \geq (1-L)(n_M-\phi-1) + (1-\alpha-L)(n_m+\phi)$, which implies $\phi \geq \frac{n_M-n_m-1}{2} + \frac{c}{2\alpha}$	e.g. from (MA, mA) to (m, A). [The altenative link, say (MA, mA) to (M, B), has weight at least $\frac{n_m-1}{2} + \frac{1-\alpha-L}{2(1-L)}n_M$.]	ϕ_{um}
segregation to M-assimilation	$(1-L)(n_M+\phi) - c \geq (1-L)(n_m-\phi-1)$, which implies $\phi \geq \frac{n_m-n_M-1}{2} + \frac{c}{2(1-L)}$	e.g. from (MA, mB) to (M, A).	ϕ_{sM}
segregation to m-assimilation	$(1-L)(n_m+\phi) - c \geq (1-L)(n_M-\phi-1)$, which implies $\phi \geq \frac{n_M-n_m-1}{2} + \frac{c}{2(1-L)}$	e.g. from (MA, mB) to (m, B).	ϕ_{sm}

Table B1 continued

M-assimilation to m-assimilation	$(1-L)\phi+(1-\alpha-L)(n-\phi-1)\geq$ $(1-L)(n-\phi-1)+(1-\alpha-L)\phi-c$; and $(1-L)(n_m+\phi')+(1-\alpha-L)(n_M-\phi'-1)-c\geq$ $(1-L)(n_M-\phi'-1)+(1-\alpha-L)(n_m+\phi')$.	e.g. from (M,A) to (m,A). Require ϕ_{Mu} mistakes for m to switch first. Then M will switch if ϕ_{um} mistakes are made.	This is minimum of $\max\{\phi_{Mu},\phi_{um}\}$ and
	$(1-L)\phi\geq(1-L)(n-\phi-1)-c$; and $(1-L)(n_m+\phi')-c\geq(1-L)(n_M-\phi'-1)$.	Alternatively examine e.g. (M,A) to (m,B). Require ϕ_{Ms} mistakes for m to switch first. Then M switch if ϕ_{sm} errors are made.	$\max\{\phi_{Ms},\phi_{sm}\}$.
m-assimilation to M-assimilation	$(1-L)\phi+(1-\alpha-L)(n-\phi-1)\geq$ $(1-L)(n-\phi-1)+(1-\alpha-L)\phi-c$; and $(1-L)(n_M+\phi')+(1-\alpha-L)(n_m-\phi'-1)-c\geq$ $(1-L)(n_m-\phi'-1)+(1-\alpha-L)(n_M+\phi')$.	e.g. from (m,A) to (M,A). ϕ_{Mu} mistakes required for M to switch first. Then m will switch if ϕ_{uM} errors are made.	This is minimum of $\max\{\phi_{Mu},\phi_{uM}\}$ and
	$(1-L)\phi\geq(1-L)(n-\phi-1)-c$; and $(1-L)(n_M+\phi')-c\geq(1-L)(n_m-\phi'-1)$.	Alternatively examine e.g. (m,A) to (M,B). ϕ_{Ms} mistakes required for M to switch first. Then m will switch if ϕ_{sM} errors are made.	$\max\{\phi_{Ms},\phi_{sM}\}$.
segregation to multiculturalism	$(1-L)\phi+(1-\alpha-L)n_M\geq(1-L)(n_m-\phi-1)$, which implies $\phi\geq\frac{n_m-1}{2}-\frac{1-\alpha-L}{2(1-L)}n_M$; and $(1-L)\phi+(1-\alpha-L)n_M\geq(1-L)n_M+(1-\alpha-L)\phi-c$ which implies $\phi\geq n_M-\frac{c}{\alpha}$.	e.g. from (MA,mB) to (MA,mA). The second inequality ensures m do not want to play (M,A).	
	$(1-L)\phi+(1-\alpha-L)n_m\geq(1-L)(n_M-\phi-1)$, which implies $\phi\geq\frac{n_M-1}{2}-\frac{1-\alpha-L}{2(1-L)}n_m$	Alternatively examine e.g. from (MA,mB) to (MB,mB). Observe that M do not want to play (m,B).	
multiculturalism to segregation	$(1-L)\phi\geq(1-L)(n_m-\phi-1)+(1-\alpha-L)n_M$, which implies $\phi\geq\frac{n_m-1}{2}+\frac{1-\alpha-L}{2(1-L)}n_M$; and $(1-L)\phi\geq(1-L)n_M+(1-\alpha-L)(n_m-k-1)-c$, which implies $\phi\geq\frac{1-L}{2(1-L)-\alpha}n_M+\frac{1-\alpha-L}{2(1-L)-\alpha}(n_m-1)-\frac{c}{2(1-L)-\alpha}$.	e.g. from (MA,mA) to (MA,mB). The second inequality ensures m do not want to play (M,A).	
	$(1-L)\phi\geq(1-L)(n_M-\phi-1)+(1-\alpha-L)n_m$, which implies $\phi\geq\frac{n_M-1}{2}+\frac{1-\alpha-L}{1-L}n_m$; and $(1-L)\phi\geq(1-L)n_m+(1-\alpha-L)(n_M-k-1)$, which implies $\phi\geq\frac{1-L}{2(1-L)-\alpha}n_m+\frac{1-\alpha-L}{2(1-L)-\alpha}(n_M-1)$.	Alternatively examine e.g. from (MA,mA) to (MB,mA). The second inequality ensures M do not want to play (m,A).	

Comments: When $\alpha>L$ for $\phi_{Mu}<\phi_{uM}$, the weight for a link from segregation to multiculturalism is lower than specified above. This does not change the results. Multiculturalism to segregation transits continue to require at least $\frac{n_m-1}{2}+\frac{1-\alpha-L}{2(1-L)}n_M$ mistakes.

indexes the set. All equilibria in a set, $\sigma \in \Sigma_x$, have the same stochastic potential.

Lemma 2 continues to apply. Notation is simplified in the same way as the proof for $1-\alpha < L$. Table B1 finds the lowest weight link between sets i.e. from some node in set Σ_x to some node in another set Σ_y.

Multiculturalism has lower stochastic potential than M-assimilation when $\phi_{Mu} < \phi_{uM}$.
Take the lowest weight tree to an M-assimilation equilibrium. The lowest weight link from this node to some multiculturalism equilibrium is ϕ_{Mu}. It can be seen from Table B1 that the weight of any link from multiculturalism to M-assimilation, m-assimilation or segregation is at least the minimum of ϕ_{uM} and $\frac{n_m-1}{2} + \frac{1-\alpha-L}{2(1-L)}n_M$. Observe $\phi_{uM} + \phi_{Mu} = n_m - 1$ and so $\phi_{Mu} < \frac{n_m-1}{2}$. Thus the result follows from Lemmas 2 and 3.

Multiculturalism has lower stochastic potential than m-assimilation when $\phi_{Mu} < \phi_{uM}$.
The transit from m-assimilation to multiculturalism also has weight ϕ_{Mu} so the argument follows as above.

M-assimilation has lower stochastic potential than multiculturalism when $\phi_{uM} < \phi_{Mu}$.
Observe that ϕ_{uM} is the lowest weight transition from multiculturalism to M-assimilation when $\phi_{uM} < \phi_{Mu}$ since then $\phi_{uM} < \frac{n_m-1}{2}$. From Table B1, any transition from M-assimilation to multiculturalism, segregation or m-assimilation has weight at least ϕ_{Mu} (since $\phi_{Ms} > \phi_{Mu}$). The result follows from Lemmas 2 and 3.

M-assimilation has lower stochastic potential than segregation when $\phi_{uM} < \phi_{Mu}$.
The transit from segregation to M-assimilation has lower weight than ϕ_{uM} and so the result follows from above.

Multiculturalism has lower stochastic potential than segregation when $\phi_{Mu} < \phi_{uM}$.
The inequality $\phi_{Mu} < \phi_{uM}$ implies $\frac{n-1}{2} - \frac{c}{2\alpha} < \frac{n_m-n_M-1}{2} + \frac{c}{2\alpha}$ which implies $n_M < \frac{c}{\alpha}$. Thus the lowest weight link from segregation to some multiculturalism equilibrium requires $\frac{n_m-1}{2} - \frac{1-\alpha-L}{2(1-L)}n_M$ mistakes. It can be seen from Table B1 that any link out of multiculturalism is of weight at least the minimum of ϕ_{uM} and $\frac{n_m-1}{2} + \frac{1-\alpha-L}{2(1-L)}n_M$.

M-assimilation has lower stochastic potential than m-assimilation when $\phi_{uM} < \phi_{Mu}$ and $\phi_{sm} > \phi_{Ms}$.

Observe $\phi_{um} > \phi_{sm} > \phi_{Ms} > \phi_{Mu} > \phi_{uM}$. Since $\phi_{Mu} > \phi_{uM}$ then $n_M > c/\alpha$ and so $\phi_{Ms} > \phi_{sM}$ but we know $\phi_{Mu} < \phi_{Ms}$ and so ϕ_{Mu} is the minimum weight transition from m-assimilation to M-assimilation.

Without loss of generality consider the tree to (m, A). Suppose the tree has a link from (M, A) to (m, A), which is of weight ϕ_{um}, or from (M, A) to (m, B), which is of weight ϕ_{sm}. Form a link from (m, A) to (M, A), of weight ϕ_{Mu}, and delete the link from (M, A) that goes to either (m, A) or (m, B), to form a tree to (M, A) with strictly lower total weight. Suppose the new tree has a link from (M, B) to (m, A), which is of weight ϕ_{sm}, or from (M, B) to (m, B), which is of weight ϕ_{um}. Delete this link and form (m, A) to (M, B), of weight ϕ_{Ms}, to form a tree to (M, B) with strictly lower total weight.

Suppose instead the tree to (m, A) has no direct links from any M-assimilation node to any m-assimilation node. Then there must be a direct link from segregation and/or multiculturalism to some m-assimilation node. Suppose the direct link is from $(MA; mA)$ to either (m, A) or (m, B). This link is of weight at least ϕ_{um} or $\frac{n_m-1}{2} + \frac{1-\alpha-L}{2(1-L)}n_M$. Delete such a link and from the same node send the link instead to (M, A) of weight $\phi_{uM} < \frac{n_m-1}{2}$.[62] Now form a link from (m, A) to (M, A) of weight ϕ_{Mu} and delete any link from (M, A) which are all of weight at least ϕ_{Mu}. Suppose the direct link is from $(MA; mB)$ to either (m, A) or (m, B). This link is of weight at least ϕ_{sm}. Delete such a link and from the same node send the link instead to (M, A) of weight ϕ_{sM}. Now form a link from (m, A) to (M, A) of weight ϕ_{Mu} and delete any link from (M, A) which are all of weight at least ϕ_{Mu}.

Finally suppose none of the other links are present in the tree to (m, A). Then there must be a link from $(mB; mA)$ or $(MB; mB)$ to either (m, A) or (m, B) or both, and no other direct links. Transfer all such links to (M, B) which, as above, produces a graph of strictly lower total weight. Every node now has a path to (M, B) apart from (m, A) (and possibly (m, B) if it links directly to (m, A)). So finally form a link from (m, A) to $(MA; mA)$ of weight ϕ_{Mu} and delete a link from (M, B) which is at least ϕ_{Mu}.

[62] Observe that the difference between ϕ_{uM} and ϕ_{um} is the same as the difference between ϕ_{sM} and ϕ_{sm}.

For $\phi \in \mathbb{R}$, under any parameters there is a unique set, Σ_x, with lowest stochastic potential. However, in a finite population ϕ is discrete. Because of this discreteness we can get intervals where the set is not unique. To avoid issues associated with discreteness we examine the parameter range $|\phi_{uM} - \phi_{Mu}| \geq 1$. Then $\phi_{uM} - \phi_{Mu} \geq 1$ implies $n_m \geq n - c/\alpha + 1$, and $\phi_{Mu} - \phi_{uM} \geq 1$ implies $n_m \leq n - c/\alpha - 1$. To avoid the same issues associated with discreteness close to parameter values $1 - L = \alpha$ we also require $\frac{n_m - 1}{2} + \frac{1-\alpha-L}{2(1-L)} n_M - \min\{\phi_{Mu}, \phi_{uM}\} \geq 1$. Since $\phi_{uM} + \phi_{Mu} = n_m - 1$ combined with the assumption $\phi_{uM} - \phi_{Mu} \geq 1$ implies $\min\{\phi_{Mu}, \phi_{uM}\} \leq \frac{n_m - 1}{2} - \frac{1}{2}$, this implies $1 - L \geq \alpha + \frac{1}{n_M - 1}$.[63] Finally Assumption 2 continues to ensure $\phi_{sm} - \phi_{Ms} \geq 1$.

[63]This also guarantees that the alternative link from segregation to mulitculturalism has weight at least one unit higher than the alternative link from multiculturalism to segregation: $\frac{n_m-1}{2} + \frac{1-\alpha-L}{2(1-L)} n_M - [\frac{n_m-1}{2} - \frac{1-\alpha-L}{2(1-L)} n_M] \geq 1$.

Online Appendix C Additional empirical results

Table C1: Number of observed cohorts by nationality

	Minimum cohort size of		
	(1) 30	(2) 20	(3) 40
Germany	303	455	234
Italy	291	404	225
Poland	196	276	149
Canada	174	259	116
Britain	167	248	127
Russia	151	219	122
Sweden	139	242	96
Ireland	133	185	109
Mexico	59	102	38
Other	133	227	91
Total	1746	2617	1307

Notes. Number of cohorts we observe, broken down by nationality. Cohorts are defined by nationality, county, tenure in the US grouped to the nearest 10 years, and year of arrival in 10 year bands. Column (1) shows the number of cohorts when we require cohorts to contain a minimum of 30 people. Columns (2) and (3) show how these results change when thresholds of 20 or 40 are used instead. Germany includes Germans and Austrians. Sweden includes Sweden, Denmark, and Norway. Other is composed (in order of size, at cohort size 30) of Romania, Japan, Netherlands, Greece, Finland, Portugal, West Indies, France, China, Spain, Belgium, Cuba, South America. These nationalities are treated as seperate from each other in our analysis, and are aggregated here only for brevity.

Table C2: Comparing the threshold effect in share speaking English, using different sample restrictions or weights

Dependent Variable: Proportion of people in the cohort that speak English
Standard Errors in Parentheses
Cohorts defined by Nationality, County, Grouped Year of Arrival, Grouped Tenure

	Minimum cohort size			
	(1) 30	(2) 20	(3) 40	(4) Weighted regression
Optimal threshold ($\beta_1 \vert \tau = \tau^*$)	-.509***	-.348***	-.475***	-.510***
	(.025)	(.016)	(.024)	(.035)
Constant (β_0)	.885***	.885***	.890***	.886***
	(.004)	(.004)	(.004)	(.004)
Optimal Threshold Level (τ^*)	.31	.27	.30	.31
F-statistic	422	457	394	208
1% critical value for F-statistic	6.6	6.6	6.6	6.6
Cohort Fixed Effects	No	No	No	No
Slopes in Nationality Share	No	No	No	No
Observations	1272	1925	955	1272

Notes. *** denotes significance at 0.1%, ** at 1%, and * at 5% level, when treated as a standalone regression. The outcome measures the proportion of the cohort that speaks English. Cohorts with English, Canadian, and Irish nationalities are excluded from the sample, since English is likely to be their mother tongue. Cohort fixed effects are composed of nationality, arrival year (grouped), and tenure (grouped) fixed effects.

All columns are regressions of share speaking English on a sequence of nationality share thresholds. We vary the threshold between .20 and .40, at intervals of .01. We provide the results for the threshold among these which produced a regression with the highest F-statistic when tested against the null of no threshold. The value for this threshold, the estimated F-statistic, and the 1% critical value for this statistic (Andrews, 1993) are provided at the bottom of the table.

Column (1) requires a minimum cohort size of 30, as in our main specifications. Columns (2) and (3) use minimum sizes of 20 and 40 respectively. Column (4) retains the minimum cohort size of 30, but first divides cohorts in the unrestricted sample into deciles based on nationality share, and then weights observations in each of these deciles in the analysis sample inversely proportionally to the size of that decile in the analysis sample.

Table C3: Comparing the threshold effect in share in-married, using different sample restrictions or weights

Dependent Variable: Proportion of married people in the cohort that are married to someone of the same nationality
Standard Errors in Parentheses
Cohorts defined by Nationality, County, Grouped Year of Arrival, Grouped Tenure

	Minimum cohort size			
	(1) 30	(2) 20	(3) 40	(4) Weighted regression
Optimal threshold ($\beta_1 \vert \tau = \tau^*$)	.151***	.094**	.128**	.142***
	(.042)	(.030)	(.048)	(.030)
Constant (β_0)	.606***	.598***	.613***	.609***
	(.006)	(.005)	(.006)	(.006)
Optimal Threshold Level (τ^*)	.35	.35	.35	.33
F-statistic	13.0	9.7	7.3	22.4
1% critical value for F-statistic	6.6	6.6	6.6	6.6
Cohort Fixed Effects	No	No	No	No
Slopes in Nationality Share	No	No	No	No
Observations	1746	2617	1307	1746

Notes. *** denotes significance at 0.1%, ** at 1%, and * at 5% level, when treated as a standalone regression. The outcome measures the married proportion of the cohort that is 'in-married' *i.e.* married to someone of the same nationality. Cohort fixed effects are composed of nationality, arrival year (grouped), and tenure (grouped) fixed effects.
All columns are regressions of share in-married on a sequence of nationality share thresholds. We vary the threshold between .20 and .40, at intervals of .01. We provide the results for the threshold among these which produced a regression with the highest F-statistic when tested against the null of no threshold. The value for this threshold, the estimated F-statistic, and the 1% critical value for this statistic (Andrews, 1993) are provided at the bottom of the table.
Column (1) requires a minimum cohort size of 30, as in our main specifications. Columns (2) and (3) use minimum sizes of 20 and 40 respectively. Column (4) retains the minimum cohort size of 30, but first divides cohorts in the unrestricted sample into deciles based on nationality share, and then weights observations in each of these deciles in the analysis sample inversely proportionally to the size of that decile in the analysis sample.